Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/167-6.2.3-e-x-^m-a+b-cosh-c+d-x^n-^p

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [68]. This is test number [167].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (68)	0.00 (0)
Mathematica	100.00 (68)	0.00 (0)
Maxima	91.18 (62)	8.82 (6)
Fricas	88.24 (60)	11.76 (8)
Maple	85.29 (58)	14.71 (10)
Giac	63.24 (43)	36.76 (25)
Sympy	35.29 (24)	64.71 (44)
Mupad	30.88 (21)	69.12 (47)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

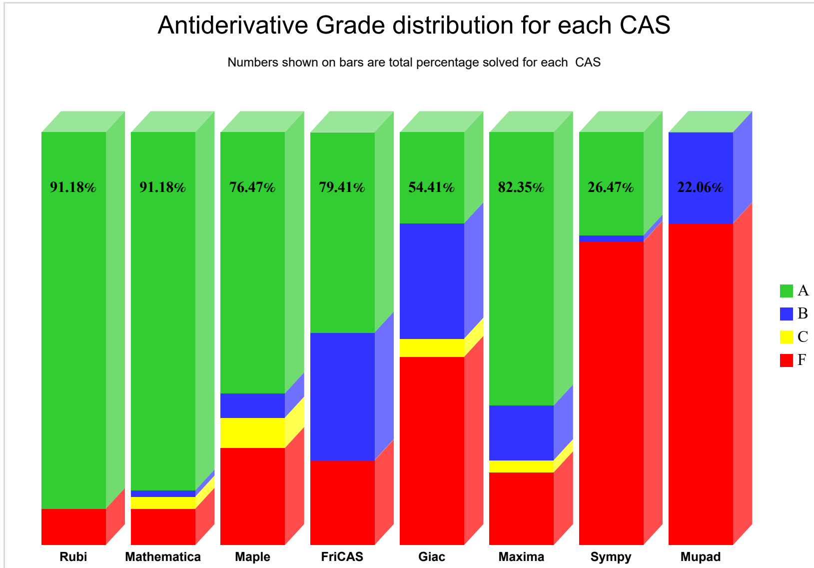
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

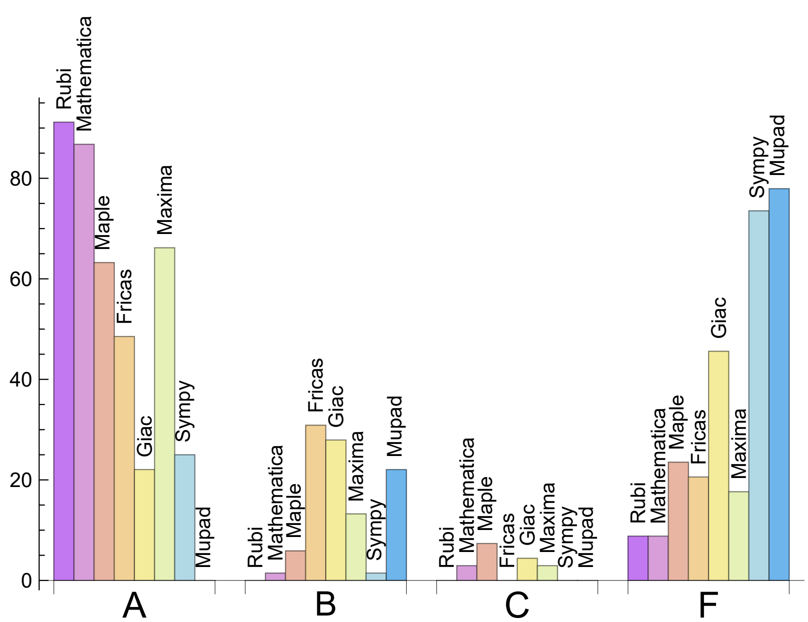
System	% A grade	% B grade	% C grade	% F grade
Mathematica	86.765	1.471	2.941	8.824
Rubi	80.882	0.000	10.294	8.824
Maxima	66.176	13.235	2.941	17.647
Maple	63.235	5.882	7.353	23.529
Fricas	48.529	30.882	0.000	20.588
Sympy	25.000	1.471	0.000	73.529
Giac	22.059	27.941	4.412	45.588
Mupad	0.000	22.059	0.000	77.941

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maxima	6	100.00	0.00	0.00
Fricas	8	100.00	0.00	0.00
Maple	10	100.00	0.00	0.00
Giac	25	100.00	0.00	0.00
Sympy	44	100.00	0.00	0.00
Mupad	47	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.25
Fricas	0.25
Maple	0.28
Rubi	0.35
Mathematica	0.47
Giac	0.66
Mupad	1.29
Sympy	3.32

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	30.14	1.00	24.00	0.89
Sympy	52.67	1.16	25.50	1.08
Mathematica	74.84	0.94	53.00	0.95
Rubi	86.24	1.01	62.50	1.00
Maxima	113.13	1.55	61.00	1.01
Maple	116.10	1.18	57.00	1.07
Fricas	149.35	1.82	70.00	1.56
Giac	165.00	1.83	65.00	1.70

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

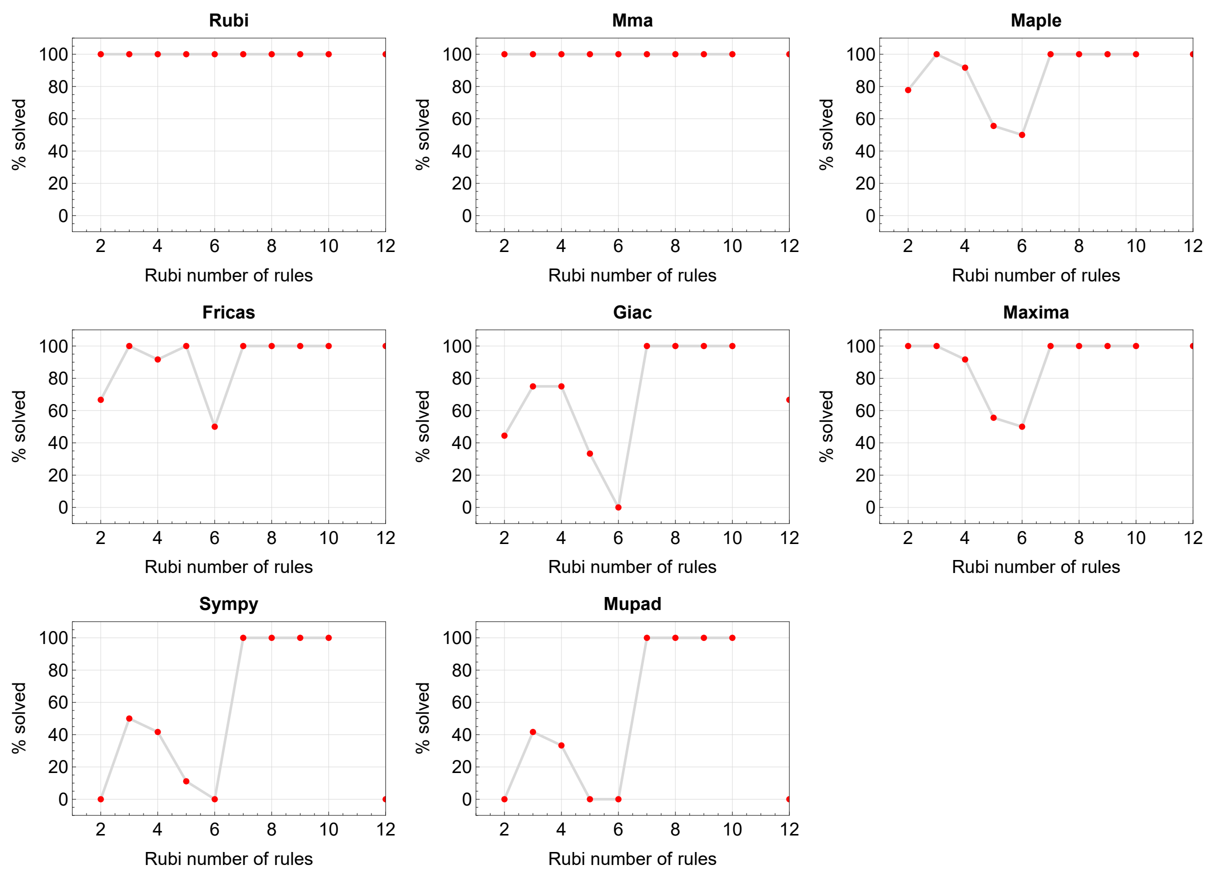


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

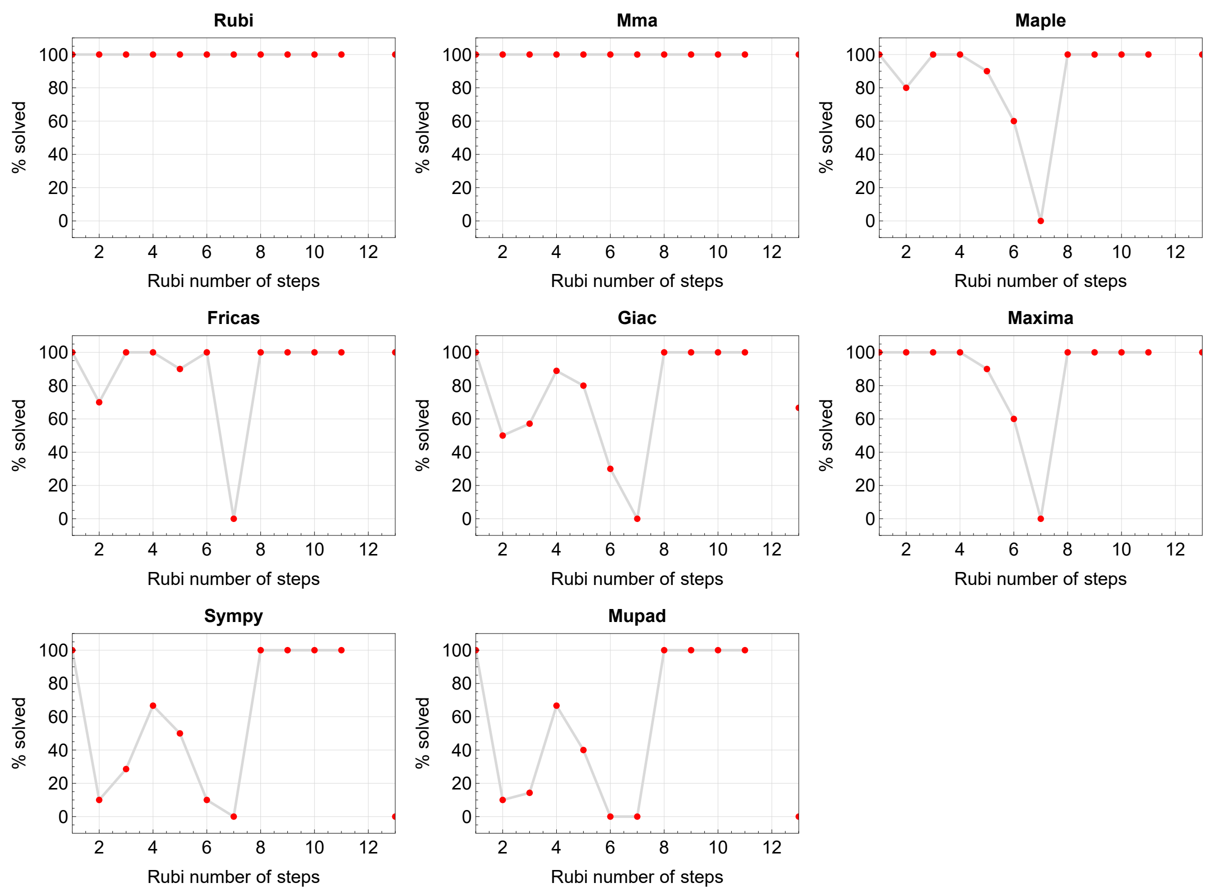


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

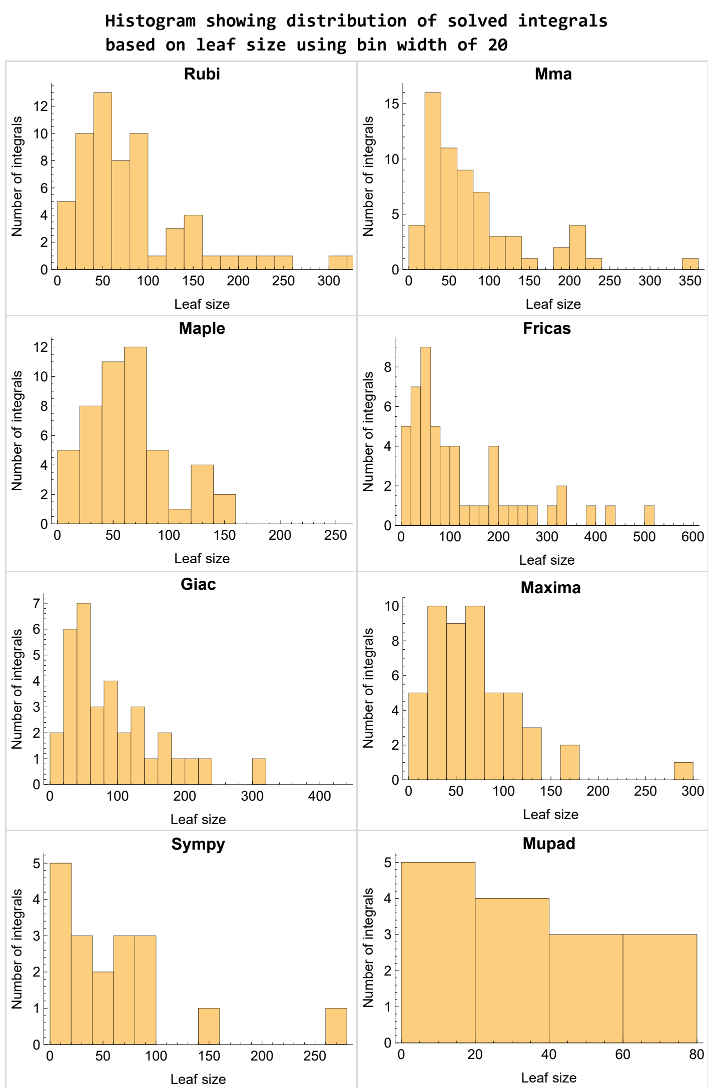


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

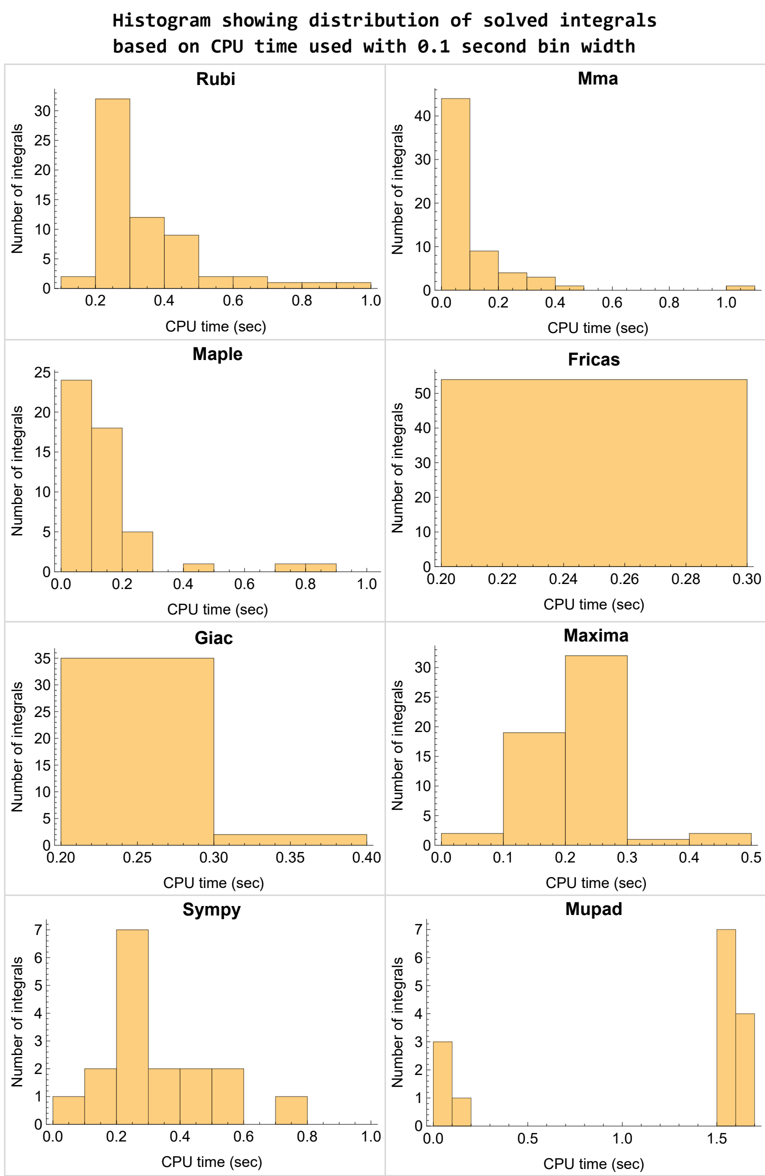


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

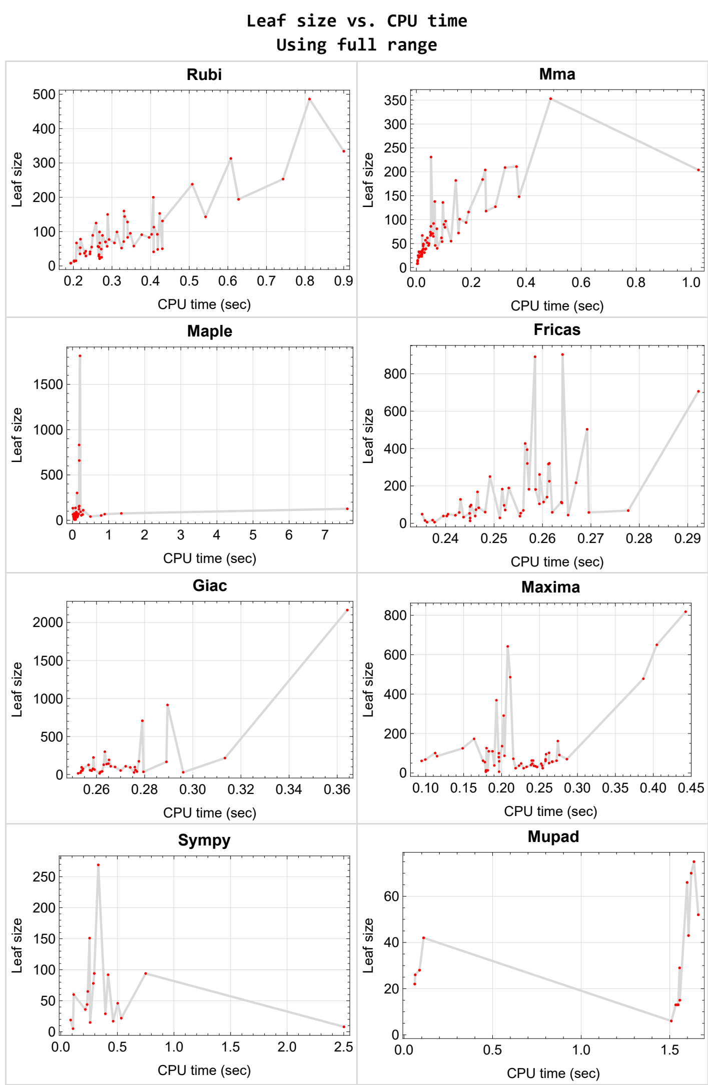


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{41, 42, 44, 46, 57, 58\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{17, 22\}$

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

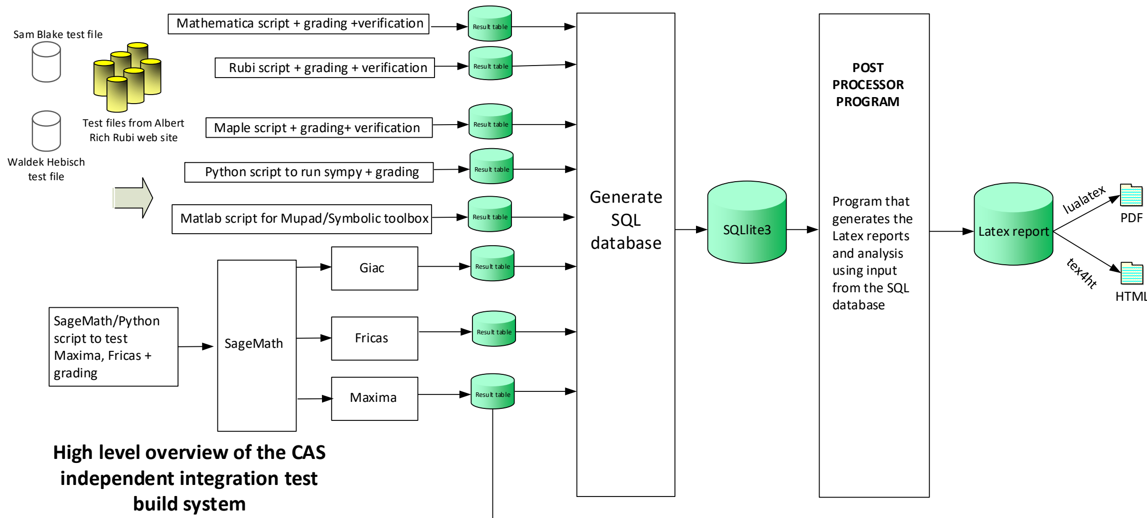
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 23, 24, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 45, 47, 48, 49, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 64, 65, 67, 68 }

B grade { }

C grade { 7, 17, 22, 25, 29, 50, 66 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 64, 65, 66 }

B grade { 3 }

C grade { 67, 68 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 38, 40, 50, 51, 52, 53, 61, 66 }

B grade { 59, 60, 64, 65 }

C grade { 35, 47, 54, 55, 56 }

F normal fail { 37, 39, 43, 45, 48, 49, 62, 63, 67, 68 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 3, 4, 5, 7, 8, 10, 11, 12, 14, 15, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 38, 40, 53, 59, 60, 61, 64, 65, 66 }

B grade { 2, 6, 9, 13, 16, 20, 21, 22, 30, 34, 36, 50, 51, 52, 54, 55, 56, 62, 63, 67, 68 }

C grade { }

F normal fail { 35, 37, 39, 43, 45, 47, 48, 49 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 23, 24, 25, 26, 27, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 47, 48, 49, 50, 51, 52, 53, 59, 60, 64, 65, 66 }

B grade { 1, 2, 4, 17, 22, 54, 55, 56, 61 }

C grade { 28, 29 }

F normal fail { 43, 45, 62, 63, 67, 68 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { 2, 4, 5, 9, 10, 11, 12, 16, 17, 18, 19, 22, 53, 61, 66 }

B grade { 1, 3, 7, 8, 14, 15, 21, 23, 24, 25, 26, 27, 28, 29, 33, 59, 60, 64, 65 }

C grade { 54, 55, 56 }

F normal fail { 6, 13, 20, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 43, 45, 47, 48, 49, 50, 51, 52, 62, 63, 67, 68 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 3, 8, 10, 15, 17, 22, 23, 24, 27, 28, 29, 33, 61, 66 }

C grade { }

F normal fail { }

F(-1) timedout fail { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 16, 18, 19, 20, 21, 25, 26, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 43, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 59, 60, 62, 63, 64, 65, 67, 68 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 3, 8, 15, 17, 22, 23, 24, 26, 27, 28, 29, 33, 59, 60, 61, 66 }

B grade { 10 }

C grade { }

F normal fail { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 16, 18, 19, 20, 21, 25, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 43, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 62, 63, 64, 65, 67, 68 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	33	31	41	80	29	36	75	28
N.S.	1	0.97	0.91	1.21	2.35	0.85	1.06	2.21	0.82
time (sec)	N/A	0.288	0.029	0.067	0.196	0.251	0.217	0.254	0.088

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	77	67	74	110	189	0	75	0
N.S.	1	1.12	0.97	1.07	1.59	2.74	0.00	1.09	0.00
time (sec)	N/A	0.304	0.052	0.049	0.183	0.253	0.000	0.258	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	31	14	13	13	19	27	13
N.S.	1	1.00	2.07	0.93	0.87	0.87	1.27	1.80	0.87
time (sec)	N/A	0.209	0.011	0.034	0.180	0.245	0.087	0.276	1.543

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	45	40	87	49	0	41	0
N.S.	1	1.00	0.85	0.75	1.64	0.92	0.00	0.77	0.00
time (sec)	N/A	0.224	0.025	0.028	0.204	0.235	0.000	0.262	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	33	24	39	0	24	0
N.S.	1	1.00	0.92	1.32	0.96	1.56	0.00	0.96	0.00
time (sec)	N/A	0.290	0.012	0.047	0.254	0.240	0.000	0.253	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	70	70	70	55	183	0	0	0
N.S.	1	1.06	1.06	1.06	0.83	2.77	0.00	0.00	0.00
time (sec)	N/A	0.295	0.053	0.050	0.178	0.252	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	48	38	58	40	71	0	108	0
N.S.	1	1.14	0.90	1.38	0.95	1.69	0.00	2.57	0.00
time (sec)	N/A	0.439	0.032	0.058	0.239	0.252	0.000	0.272	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	55	40	46	59	60	78	142	42
N.S.	1	1.08	0.78	0.90	1.16	1.18	1.53	2.78	0.82
time (sec)	N/A	0.258	0.076	0.148	0.197	0.248	0.287	0.265	0.111

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	101	90	95	427	0	97	0
N.S.	1	1.00	1.02	0.91	0.96	4.31	0.00	0.98	0.00
time (sec)	N/A	0.282	0.157	0.104	0.259	0.256	0.000	0.276	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	35	27	24	38	28	60	54	22
N.S.	1	1.13	0.87	0.77	1.23	0.90	1.94	1.74	0.71
time (sec)	N/A	0.228	0.020	0.071	0.190	0.245	0.114	0.254	0.061

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	86	51	56	73	0	58	0
N.S.	1	1.00	1.10	0.65	0.72	0.94	0.00	0.74	0.00
time (sec)	N/A	0.225	0.053	0.065	0.267	0.246	0.000	0.276	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	34	31	49	0	35	0
N.S.	1	1.00	0.89	0.92	0.84	1.32	0.00	0.95	0.00
time (sec)	N/A	0.237	0.016	0.104	0.233	0.240	0.000	0.279	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	92	94	86	61	394	0	0	0
N.S.	1	1.05	1.07	0.98	0.69	4.48	0.00	0.00	0.00
time (sec)	N/A	0.424	0.181	0.101	0.258	0.257	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	46	66	36	90	0	126	0
N.S.	1	1.00	0.81	1.16	0.63	1.58	0.00	2.21	0.00
time (sec)	N/A	0.313	0.069	0.110	0.243	0.245	0.000	0.257	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	83	55	93	100	96	92	192	70
N.S.	1	1.05	0.70	1.18	1.27	1.22	1.16	2.43	0.89
time (sec)	N/A	0.405	0.126	0.223	0.196	0.252	0.418	0.265	1.621

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	184	157	162	903	0	166	0
N.S.	1	1.00	1.15	0.98	1.01	5.64	0.00	1.04	0.00
time (sec)	N/A	0.357	0.241	0.186	0.274	0.264	0.000	0.289	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	29	33	28	62	38	44	56	26
N.S.	1	0.88	1.00	0.85	1.88	1.15	1.33	1.70	0.79
time (sec)	N/A	0.249	0.009	0.088	0.175	0.255	0.233	0.257	0.063

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	136	86	91	113	0	95	0
N.S.	1	1.00	1.09	0.69	0.73	0.90	0.00	0.76	0.00
time (sec)	N/A	0.278	0.097	0.104	0.276	0.264	0.000	0.254	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	69	50	83	0	50	0
N.S.	1	1.00	0.89	1.25	0.91	1.51	0.00	0.91	0.00
time (sec)	N/A	0.281	0.025	0.170	0.263	0.247	0.000	0.258	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	144	204	149	102	891	0	0	0
N.S.	1	1.06	1.50	1.10	0.75	6.55	0.00	0.00	0.00
time (sec)	N/A	0.355	0.251	0.178	0.263	0.258	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	92	121	58	168	0	224	0
N.S.	1	1.00	1.01	1.33	0.64	1.85	0.00	2.46	0.00
time (sec)	N/A	0.413	0.063	0.181	0.267	0.247	0.000	0.259	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	41	67	52	126	128	94	108	52
N.S.	1	0.61	1.00	0.78	1.88	1.91	1.40	1.61	0.78
time (sec)	N/A	0.257	0.022	0.793	0.180	0.243	0.750	0.266	1.661

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	15	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	1.88	0.75
time (sec)	N/A	0.205	0.003	0.053	0.179	0.238	0.108	0.261	1.509

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	8	15	15
N.S.	1	1.00	1.00	0.88	0.75	0.75	1.00	1.88	1.88
time (sec)	N/A	0.201	0.005	0.086	0.197	0.236	2.502	0.252	1.557

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	41	33	56	37	58	0	173	0
N.S.	1	1.24	1.00	1.70	1.12	1.76	0.00	5.24	0.00
time (sec)	N/A	0.436	0.021	0.110	0.223	0.243	0.000	0.278	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	27	24	39	17	43	0
N.S.	1	1.00	1.00	1.29	1.14	1.86	0.81	2.05	0.00
time (sec)	N/A	0.287	0.008	0.075	0.218	0.240	0.462	0.254	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	15	15	29	13
N.S.	1	1.00	1.00	1.08	1.00	1.15	1.15	2.23	1.00
time (sec)	N/A	0.219	0.005	0.052	0.179	0.236	0.259	0.296	1.548

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	43	47	33	29	93	29
N.S.	1	1.00	1.00	1.48	1.62	1.14	1.00	3.21	1.00
time (sec)	N/A	0.280	0.020	0.088	0.252	0.244	0.394	0.274	1.556

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	58	39	65	48	43	46	216	66
N.S.	1	1.26	0.85	1.41	1.04	0.93	1.00	4.70	1.43
time (sec)	N/A	0.366	0.035	0.117	0.226	0.242	0.503	0.313	1.598

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	71	71	70	72	225	0	0	0
N.S.	1	1.06	1.06	1.04	1.07	3.36	0.00	0.00	0.00
time (sec)	N/A	0.351	0.060	0.079	0.215	0.261	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	33	24	39	0	0	0
N.S.	1	1.00	1.00	1.32	0.96	1.56	0.00	0.00	0.00
time (sec)	N/A	0.286	0.014	0.073	0.229	0.246	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	44	63	53	0	0	0
N.S.	1	1.00	0.86	0.77	1.11	0.93	0.00	0.00	0.00
time (sec)	N/A	0.296	0.026	0.078	0.240	0.245	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	17	22	33	13
N.S.	1	1.00	1.00	0.93	0.87	1.13	1.47	2.20	0.87
time (sec)	N/A	0.222	0.005	0.060	0.182	0.237	0.534	0.262	1.534

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	83	74	82	63	250	0	0	0
N.S.	1	1.11	0.99	1.09	0.84	3.33	0.00	0.00	0.00
time (sec)	N/A	0.355	0.051	0.112	0.242	0.249	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	62	74	61	0	0	0	0
N.S.	1	1.00	0.93	1.10	0.91	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	0.040	0.111	0.095	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	33	30	54	0	0	0
N.S.	1	1.00	0.92	1.32	1.20	2.16	0.00	0.00	0.00
time (sec)	N/A	0.277	0.020	0.130	0.247	0.255	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	81	0	68	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.76	0.00	0.00	0.00	0.00
time (sec)	N/A	0.267	0.075	0.000	0.100	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	36	40	37	69	0	0	0
N.S.	1	1.00	0.84	0.93	0.86	1.60	0.00	0.00	0.00
time (sec)	N/A	0.243	0.022	0.495	0.254	0.256	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	150	138	0	125	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.83	0.00	0.00	0.00	0.00
time (sec)	N/A	0.301	0.067	0.000	0.149	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	52	67	62	114	0	0	0
N.S.	1	1.00	0.78	1.00	0.93	1.70	0.00	0.00	0.00
time (sec)	N/A	0.277	0.039	0.891	0.273	0.260	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11
time (sec)	N/A	0.185	4.106	0.042	0.401	0.255	8.296	1.354	1.573

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.10
time (sec)	N/A	0.191	6.143	0.035	0.413	0.268	25.338	7.712	1.591

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	90	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.370	0.100	0.000	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	20	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.91	1.09	1.09
time (sec)	N/A	0.287	4.414	0.053	0.420	0.256	7.942	1.322	1.547

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	148	0	0	0	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.443	0.374	0.000	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	22	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	1.08
time (sec)	N/A	0.291	6.373	0.051	0.421	0.277	24.084	7.296	1.547

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	84	110	85	0	0	0	0
N.S.	1	1.00	0.94	1.24	0.96	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	0.104	0.295	0.115	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	116	0	101	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.79	0.00	0.00	0.00	0.00
time (sec)	N/A	0.360	0.190	0.000	0.112	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	200	200	182	0	173	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.86	0.00	0.00	0.00	0.00
time (sec)	N/A	0.427	0.144	0.000	0.164	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	50	46	63	34	140	0	0	0
N.S.	1	1.11	1.02	1.40	0.76	3.11	0.00	0.00	0.00
time (sec)	N/A	0.459	0.044	0.281	0.245	0.261	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	75	47	182	0	0	0
N.S.	1	1.00	0.81	1.12	0.70	2.72	0.00	0.00	0.00
time (sec)	N/A	0.329	0.094	1.352	0.240	0.257	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	97	126	70	320	0	0	0
N.S.	1	1.00	0.86	1.12	0.62	2.83	0.00	0.00	0.00
time (sec)	N/A	0.438	0.107	7.612	0.286	0.257	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	70	57	54	69	98	0	52	0
N.S.	1	0.99	0.80	0.76	0.97	1.38	0.00	0.73	0.00
time (sec)	N/A	0.297	0.033	0.244	0.258	0.245	0.000	0.270	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	99	62	136	818	321	0	137	0
N.S.	1	0.88	0.55	1.20	7.24	2.84	0.00	1.21	0.00
time (sec)	N/A	0.325	0.091	0.081	0.443	0.261	0.000	0.264	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	49	39	66	650	261	0	99	0
N.S.	1	0.91	0.72	1.22	12.04	4.83	0.00	1.83	0.00
time (sec)	N/A	0.288	0.025	0.050	0.405	0.259	0.000	0.268	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	35	25	36	478	59	0	39	0
N.S.	1	0.95	0.68	0.97	12.92	1.59	0.00	1.05	0.00
time (sec)	N/A	0.250	0.006	0.029	0.387	0.262	0.000	0.277	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	23	20	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.92	1.67	1.17	1.17
time (sec)	N/A	0.254	1.091	0.020	0.846	0.246	3.020	0.273	1.651

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	23	22	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.92	1.83	1.17	1.17
time (sec)	N/A	0.256	3.360	0.020	0.610	0.251	3.036	0.271	1.572

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	313	209	831	486	104	269	915	0
N.S.	1	0.90	0.60	2.40	1.40	0.30	0.78	2.64	0.00
time (sec)	N/A	0.652	0.323	0.183	0.211	0.259	0.331	0.289	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	153	72	301	291	68	151	300	0
N.S.	1	0.92	0.43	1.80	1.74	0.41	0.90	1.80	0.00
time (sec)	N/A	0.458	0.153	0.125	0.203	0.278	0.254	0.263	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	52	50	63	110	44	65	65	43
N.S.	1	0.96	0.93	1.17	2.04	0.81	1.20	1.20	0.80
time (sec)	N/A	0.347	0.046	0.011	0.188	0.265	0.239	0.259	1.606

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	143	127	0	0	217	0	0	0
N.S.	1	1.15	1.02	0.00	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.581	0.288	0.000	0.000	0.267	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	182	194	204	0	0	317	0	0	0
N.S.	1	1.07	1.12	0.00	0.00	1.74	0.00	0.00	0.00
time (sec)	N/A	0.662	1.027	0.000	0.000	0.261	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	537	486	353	1815	642	181	0	2163	0
N.S.	1	0.91	0.66	3.38	1.20	0.34	0.00	4.03	0.00
time (sec)	N/A	0.880	0.489	0.207	0.208	0.259	0.000	0.364	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	238	118	659	369	109	0	707	0
N.S.	1	0.91	0.45	2.52	1.41	0.42	0.00	2.71	0.00
time (sec)	N/A	0.538	0.254	0.186	0.193	0.264	0.000	0.279	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	92	65	133	136	58	94	128	75
N.S.	1	1.08	0.76	1.56	1.60	0.68	1.11	1.51	0.88
time (sec)	N/A	0.445	0.062	0.009	0.201	0.270	0.296	0.263	1.637

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	232	253	231	0	0	503	0	0	0
N.S.	1	1.09	1.00	0.00	0.00	2.17	0.00	0.00	0.00
time (sec)	N/A	0.801	0.053	0.000	0.000	0.269	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	329	334	211	0	0	706	0	0	0
N.S.	1	1.02	0.64	0.00	0.00	2.15	0.00	0.00	0.00
time (sec)	N/A	0.973	0.364	0.000	0.000	0.292	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [25] had the largest ratio of [1.50000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	7	0.97	12	0.583
2	A	4	4	1.12	12	0.333
3	A	4	3	1.00	10	0.300
4	A	3	3	1.00	8	0.375
5	A	3	3	1.00	12	0.250
6	A	4	4	1.06	12	0.333
7	C	13	12	1.14	12	1.000
8	A	5	4	1.08	14	0.286
9	A	2	2	1.00	14	0.143
10	A	5	4	1.13	12	0.333
11	A	2	2	1.00	10	0.200
12	A	2	2	1.00	14	0.143
13	A	6	6	1.05	14	0.429
14	A	2	2	1.00	14	0.143
15	A	10	9	1.05	14	0.643
16	A	2	2	1.00	14	0.143
17	C	5	4	0.88	12	0.333
18	A	2	2	1.00	10	0.200
19	A	2	2	1.00	14	0.143
20	A	3	3	1.06	14	0.214
21	A	2	2	1.00	14	0.143
22	C	5	4	0.61	12	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	4	3	1.00	8	0.375
24	A	4	3	1.00	8	0.375
25	C	13	12	1.24	8	1.500
26	A	3	3	1.00	12	0.250
27	A	4	3	1.00	12	0.250
28	A	8	7	1.00	12	0.583
29	C	10	9	1.26	12	0.750
30	A	6	5	1.06	8	0.625
31	A	3	3	1.00	12	0.250
32	A	5	4	1.00	12	0.333
33	A	4	3	1.00	12	0.250
34	A	6	5	1.11	12	0.417
35	A	2	2	1.00	8	0.250
36	A	3	3	1.00	12	0.250
37	A	2	2	1.00	10	0.200
38	A	2	2	1.00	14	0.143
39	A	2	2	1.00	10	0.200
40	A	2	2	1.00	14	0.143
41	N/A	1	0	1.00	18	0.000
42	N/A	1	0	1.00	20	0.000
43	A	5	4	1.00	20	0.200
44	N/A	2	0	1.00	22	0.000
45	A	7	6	1.00	22	0.273
46	N/A	2	0	1.00	24	0.000
47	A	2	2	1.00	12	0.167
48	A	2	2	1.00	14	0.143
49	A	2	2	1.00	14	0.143
50	C	13	12	1.11	16	0.750
51	A	2	2	1.00	18	0.111
52	A	2	2	1.00	18	0.111
53	A	5	4	0.99	18	0.222
54	A	4	3	0.88	12	0.250
55	A	5	4	0.91	10	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	5	4	0.95	8	0.500
57	N/A	4	0	1.00	12	0.000
58	N/A	3	0	1.00	12	0.000
59	A	5	4	0.90	18	0.222
60	A	6	5	0.92	16	0.312
61	A	9	8	0.96	14	0.571
62	A	6	5	1.15	18	0.278
63	A	6	5	1.07	18	0.278
64	A	6	5	0.91	18	0.278
65	A	6	5	0.91	16	0.312
66	C	11	10	1.08	14	0.714
67	A	6	5	1.09	18	0.278
68	A	6	5	1.02	18	0.278

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int x^3 \cosh(a + bx^2) \, dx$

3.1.1 Optimal result

Integrand size = 12, antiderivative size = 34

$$\int x^3 \cosh(a + bx^2) \, dx = -\frac{\cosh(a + bx^2)}{2b^2} + \frac{x^2 \sinh(a + bx^2)}{2b}$$

output
```
-1/2*cosh(b*x^2+a)/b^2+1/2*x^2*sinh(b*x^2+a)/b
```

3.1.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int x^3 \cosh(a + bx^2) \, dx = \frac{-\cosh(a + bx^2) + bx^2 \sinh(a + bx^2)}{2b^2}$$

input
```
Integrate[x^3*Cosh[a + b*x^2],x]
```

output
```
(-Cosh[a + b*x^2] + b*x^2*Sinh[a + b*x^2])/(2*b^2)
```

3.1.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5844, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cosh \left(a+b x^2\right) \, dx$$

$$\Big\downarrow \ 5844$$

$$\frac{1}{2} \int x^2 \cosh \left(b x^2+a\right)dx^2$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2} \int x^2 \sin \left(i b x^2+i a+\frac{\pi}{2}\right)dx^2$$

$$\Big\downarrow \ 3777$$

$$\frac{1}{2} \left(\frac{x^2 \sinh \left(a+b x^2\right)}{b}-\frac{i \int -i \sinh \left(b x^2+a\right)dx^2}{b}\right)$$

$$\Big\downarrow \ 26$$

$$\frac{1}{2} \left(\frac{x^2 \sinh \left(a+b x^2\right)}{b}-\frac{\int \sinh \left(b x^2+a\right)dx^2}{b}\right)$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2} \left(\frac{x^2 \sinh \left(a+b x^2\right)}{b}-\frac{\int -i \sin \left(i b x^2+i a\right)dx^2}{b}\right)$$

$$\Big\downarrow \ 26$$

$$\frac{1}{2} \left(\frac{x^2 \sinh \left(a+b x^2\right)}{b}+\frac{i \int \sin \left(i b x^2+i a\right)dx^2}{b}\right)$$

$$\Big\downarrow \ 3118$$

$$\frac{1}{2} \left(\frac{x^2 \sinh \left(a+b x^2\right)}{b}-\frac{\cosh \left(a+b x^2\right)}{b^2}\right)$$

input
```
Int[x^3*Cosh[a + b*x^2],x]
```

output
```
(-(Cosh[a + b*x^2]/b^2) + (x^2*Sinh[a + b*x^2])/b)/2
```

3.1.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

3.1.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

method	result	size
parallelrisch	$\frac{1-\tanh\left(\frac{b\,x^{2}}{2}+\frac{a}{2}\right)x^{2}b}{b^{2}\left(\tanh\left(\frac{b\,x^{2}}{2}+\frac{a}{2}\right)^{2}-1\right)}$	41
risch	$\frac{(b\,x^{2}-1)\,{\mathrm e}^{b\,x^{2}+a}}{4b^{2}}-\frac{(b\,x^{2}+1)\,{\mathrm e}^{-b\,x^{2}-a}}{4b^{2}}$	45
meijerg	$-\frac{\cosh(a)\sqrt{\pi}\,\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh\left(b\,x^{2}\right)}{2\sqrt{\pi}}-\frac{b\,x^{2}\sinh\left(b\,x^{2}\right)}{2\sqrt{\pi}}\right)}{b^{2}}+\frac{\sinh(a)\left(\cosh\left(b\,x^{2}\right)b\,x^{2}-\sinh\left(b\,x^{2}\right)\right)}{2b^{2}}$	71

input
```
int(x^3*cosh(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
(1-tanh(1/2*b*x^2+1/2*a)*x^2*b)/b^2/(tanh(1/2*b*x^2+1/2*a)^2-1)
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int x^3 \cosh \left(a + bx^2\right) \, dx = \frac{bx^2 \sinh \left(bx^2 + a\right) - \cosh \left(bx^2 + a\right)}{2 \, b^2}$$

input
```
integrate(x^3*cosh(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(b*x^2*sinh(b*x^2 + a) - cosh(b*x^2 + a))/b^2
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int x^3 \cosh \left(a + bx^2\right) \, dx = \begin{cases} \frac{x^2 \sinh \left(a+bx^2\right)}{2b} - \frac{\cosh \left(a+bx^2\right)}{2b^2} & \text{for } b \neq 0 \\ \frac{x^4 \cosh \left(a\right)}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*cosh(b*x**2+a),x)
```

output
```
Piecewise((x**2*sinh(a + b*x**2)/(2*b) - cosh(a + b*x**2)/(2*b**2), Ne(b,
0)), (x**4*cosh(a)/4, True))
```

3.1.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(30) = 60.

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35

$$\int x^3 \cosh \left(a + bx^2\right) \, dx$$
$$= \frac{1}{4} \, x^4 \cosh \left(bx^2 + a\right) - \frac{1}{8} \, b \left(\frac{\left(b^2 x^4 e^a - 2 \, bx^2 e^a + 2 \, e^a\right) e^{\left(bx^2\right)}}{b^3} + \frac{\left(b^2 x^4 + 2 \, bx^2 + 2\right) e^{\left(-bx^2 - a\right)}}{b^3} \right)$$

input
```
integrate(x^3*cosh(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*x^4*cosh(b*x^2 + a) - 1/8*b*((b^2*x^4*e^a - 2*b*x^2*e^a + 2*e^a)*e^(b*
x^2)/b^3 + (b^2*x^4 + 2*b*x^2 + 2)*e^(-b*x^2 - a)/b^3)
```

3.1.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(30) = 60$.

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int x^3 \cosh\left(a + bx^2\right) dx = \frac{(bx^2 + a - 1)e^{(bx^2+a)} - (bx^2 + a + 1)e^{(-bx^2-a)}}{4\,b^2} - \frac{ae^{(bx^2+a)} - ae^{(-bx^2-a)}}{4\,b^2}$$

input
```
integrate(x^3*cosh(b*x^2+a),x, algorithm="giac")
```

output
```
1/4*((b*x^2 + a - 1)*e^(b*x^2 + a) - (b*x^2 + a + 1)*e^(-b*x^2 - a))/b^2 -
 1/4*(a*e^(b*x^2 + a) - a*e^(-b*x^2 - a))/b^2
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int x^3 \cosh\left(a + bx^2\right) dx = -\frac{\cosh(b\,x^2 + a) - b\,x^2 \sinh(b\,x^2 + a)}{2\,b^2}$$

input
```
int(x^3*cosh(a + b*x^2),x)
```

output
```
-(cosh(a + b*x^2) - b*x^2*sinh(a + b*x^2))/(2*b^2)
```

3.2 $\int x^2 \cosh(a + bx^2)\, dx$

3.2.1 Optimal result

Integrand size = 12, antiderivative size = 69

$$\int x^2 \cosh(a + bx^2)\, dx = \frac{e^{-a}\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}x\right)}{8b^{3/2}} - \frac{e^{a}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}x\right)}{8b^{3/2}} + \frac{x\sinh(a + bx^2)}{2b}$$

output
```
1/2*x*sinh(b*x^2+a)/b+1/8*erf(x*b^(1/2))*Pi^(1/2)/b^(3/2)/exp(a)-1/8*exp(a
)*erfi(x*b^(1/2))*Pi^(1/2)/b^(3/2)
```

3.2.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int x^2 \cosh(a + bx^2)\, dx$$
$$= \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}x\right)(\cosh(a) - \sinh(a)) - \sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}x\right)(\cosh(a) + \sinh(a)) + 4\sqrt{b}x\sinh(a + bx^2)}{8b^{3/2}}$$

input
```
Integrate[x^2*Cosh[a + b*x^2],x]
```

output
```
(Sqrt[Pi]*Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) - Sqrt[Pi]*Erfi[Sqrt[b]*x]*(C
osh[a] + Sinh[a]) + 4*Sqrt[b]*x*Sinh[a + b*x^2])/(8*b^(3/2))
```

3.2.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5848, 5821, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cosh\left(a+bx^2\right) \, dx$$

$$\Big\downarrow \text{ 5848}$$

$$\frac{x \sinh\left(a+bx^2\right)}{2b}-\frac{\int \sinh\left(bx^2+a\right)dx}{2b}$$

$$\Big\downarrow \text{ 5821}$$

$$\frac{x \sinh\left(a+bx^2\right)}{2b}-\frac{\frac{1}{2}\int e^{bx^2+a}dx-\frac{1}{2}\int e^{-bx^2-a}dx}{2b}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{x \sinh\left(a+bx^2\right)}{2b}-\frac{\frac{\sqrt{\pi}e^a \text{erfi}\left(\sqrt{b}x\right)}{4\sqrt{b}}-\frac{1}{2}\int e^{-bx^2-a}dx}{2b}$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{x \sinh\left(a+bx^2\right)}{2b}-\frac{\frac{\sqrt{\pi}e^a \text{erfi}\left(\sqrt{b}x\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}e^{-a} \text{erf}\left(\sqrt{b}x\right)}{4\sqrt{b}}}{2b}$$

input
```
Int[x^2*Cosh[a + b*x^2],x]
```

output
```
-1/2*(-1/4*(Sqrt[Pi]*Erf[Sqrt[b]*x])/(Sqrt[b]*E^a) + (E^a*Sqrt[Pi]*Erfi[Sq
rt[b]*x])/(4*Sqrt[b]))/b + (x*Sinh[a + b*x^2])/(2*b)
```

3.2.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 5821
```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[1/2   Int[E^(c + d*x^n
), x], x] - Simp[1/2   Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IG
tQ[n, 1]
```

rule 5848
```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1
)/(d*n))   Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[0, n, m + 1]
```

3.2.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{e^{-a} x\, e^{-b\,x^2}}{4b}+\frac{\operatorname{erf}\left(x\sqrt{b}\right)\sqrt{\pi}\, e^{-a}}{8b^{\frac{3}{2}}}+\frac{e^{a} e^{b\,x^2} x}{4b}-\frac{e^{a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b}\,x\right)}{8b\sqrt{-b}}$
meijerg	$-\frac{i\cosh(a)\sqrt{\pi}\,\sqrt{2}\left(\frac{x\sqrt{2}\,(ib)^{\frac{3}{2}} e^{b\,x^2}}{4\sqrt{\pi}\,b}-\frac{x\sqrt{2}\,(ib)^{\frac{3}{2}} e^{-b\,x^2}}{4\sqrt{\pi}\,b}+\frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\operatorname{erf}\left(x\sqrt{b}\right)}{8b^{\frac{3}{2}}}-\frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\operatorname{erfi}\left(x\sqrt{b}\right)}{8b^{\frac{3}{2}}}\right)}{2b\sqrt{ib}}-\frac{\sinh(a)\sqrt{\pi}\,\sqrt{2}\left(\frac{x\sqrt{2}\,(ib)^{\frac{5}{2}} e^{-b\,x^2}}{4\sqrt{\pi}\,b^2}\right.}{}$

input
```
int(x^2*cosh(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/exp(a)/b*x*exp(-b*x^2)+1/8*erf(x*b^(1/2))*Pi^(1/2)/b^(3/2)/exp(a)+1/4
*exp(a)*exp(b*x^2)*x/b-1/8*exp(a)/b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

3.2.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(49) = 98.

Time = 0.25 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.74

$$\int x^2 \cosh\left(a + bx^2\right)\,dx$$
$$= \frac{2\,bx\cosh\left(bx^2+a\right)^2 + 4\,bx\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right) + 2\,bx\sinh\left(bx^2+a\right)^2 + \sqrt{\pi}(\cosh\left(bx^2+a\right)\cosh\left(\right.}{}$$

input
```
integrate(x^2*cosh(b*x^2+a),x, algorithm="fricas")
```

output
```
1/8*(2*b*x*cosh(b*x^2 + a)^2 + 4*b*x*cosh(b*x^2 + a)*sinh(b*x^2 + a) + 2*b
*x*sinh(b*x^2 + a)^2 + sqrt(pi)*(cosh(b*x^2 + a)*cosh(a) + (cosh(a) + sinh
(a))*sinh(b*x^2 + a) + cosh(b*x^2 + a)*sinh(a))*sqrt(-b)*erf(sqrt(-b)*x) +
 sqrt(pi)*(cosh(b*x^2 + a)*cosh(a) + (cosh(a) - sinh(a))*sinh(b*x^2 + a) -
 cosh(b*x^2 + a)*sinh(a))*sqrt(b)*erf(sqrt(b)*x) - 2*b*x)/(b^2*cosh(b*x^2
+ a) + b^2*sinh(b*x^2 + a))
```

3.2.6 Sympy [F]

$$\int x^2 \cosh\left(a + bx^2\right)\,dx = \int x^2 \cosh\left(a + bx^2\right)dx$$

input
```
integrate(x**2*cosh(b*x**2+a),x)
```

output
```
Integral(x**2*cosh(a + b*x**2), x)
```

3.2.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(49) = 98.

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.59

$$\int x^2 \cosh\left(a + bx^2\right)\,dx = \frac{1}{3}x^3\cosh\left(bx^2 + a\right)$$
$$-\frac{1}{24}b\left(\frac{2\left(2\,bx^3e^a - 3\,xe^a\right)e^{\left(bx^2\right)}}{b^2} + \frac{2\left(2\,bx^3 + 3\,x\right)e^{\left(-bx^2-a\right)}}{b^2} - \frac{3\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}x\right)e^{(-a)}}{b^{\frac{5}{2}}} + \frac{3\sqrt{\pi}\operatorname{erf}\left(\sqrt{-b}x\right)e}{\sqrt{-b}b^2}\right.$$

input
```
integrate(x^2*cosh(b*x^2+a),x, algorithm="maxima")
```

output
```
1/3*x^3*cosh(b*x^2 + a) - 1/24*b*(2*(2*b*x^3*e^a - 3*x*e^a)*e^(b*x^2)/b^2
+ 2*(2*b*x^3 + 3*x)*e^(-b*x^2 - a)/b^2 - 3*sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/
b^(5/2) + 3*sqrt(pi)*erf(sqrt(-b)*x)*e^a/(sqrt(-b)*b^2))
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int x^2 \cosh \left(a+bx^2\right) \, dx = \frac{xe^{\left(bx^2+a\right)}}{4\,b} - \frac{xe^{\left(-bx^2-a\right)}}{4\,b} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{b}x\right)e^{(-a)}}{8\,b^{\frac{3}{2}}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-b}x\right)e^{a}}{8\,\sqrt{-b}b}$$

input
```
integrate(x^2*cosh(b*x^2+a),x, algorithm="giac")
```

output
```
1/4*x*e^(b*x^2 + a)/b - 1/4*x*e^(-b*x^2 - a)/b - 1/8*sqrt(pi)*erf(-sqrt(b)
*x)*e^(-a)/b^(3/2) + 1/8*sqrt(pi)*erf(-sqrt(-b)*x)*e^a/(sqrt(-b)*b)
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh \left(a+bx^2\right) \, dx = \int x^2 \cosh\left(b\,x^2+a\right) \, dx$$

input
```
int(x^2*cosh(a + b*x^2),x)
```

output
```
int(x^2*cosh(a + b*x^2), x)
```

3.3 $\int x \cosh(a + bx^2)\, dx$

3.3.1 Optimal result

Integrand size = 10, antiderivative size = 15

$$\int x \cosh(a + bx^2)\, dx = \frac{\sinh(a + bx^2)}{2b}$$

output
```
1/2*sinh(b*x^2+a)/b
```

3.3.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int x \cosh(a + bx^2)\, dx = \frac{\cosh(bx^2)\sinh(a)}{2b} + \frac{\cosh(a)\sinh(bx^2)}{2b}$$

input
```
Integrate[x*Cosh[a + b*x^2],x]
```

output
```
(Cosh[b*x^2]*Sinh[a])/(2*b) + (Cosh[a]*Sinh[b*x^2])/(2*b)
```

3.3.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5844, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cosh \left(a+b x^2\right) \, dx$$

$$\Big\downarrow \text{5844}$$

$$\frac{1}{2} \int \cosh \left(b x^2+a\right) dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2} \int \sin \left(i b x^2+i a+\frac{\pi}{2}\right) dx^2$$

$$\Big\downarrow \text{3117}$$

$$\frac{\sinh \left(a+b x^2\right)}{2 b}$$

input
```
Int[x*Cosh[a + b*x^2],x]
```

output
```
Sinh[a + b*x^2]/(2*b)
```

3.3.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

3.3.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sinh(b\,x^2+a)}{2b}$	14
default	$\frac{\sinh(b\,x^2+a)}{2b}$	14
parallelrisch	$\frac{\sinh(b\,x^2+a)}{2b}$	14
risch	$\frac{e^{b\,x^2+a}}{4b}-\frac{e^{-b\,x^2-a}}{4b}$	31
meijerg	$\frac{\cosh(a)\sinh(b\,x^2)}{2b}-\frac{\sinh(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(b\,x^2)}{\sqrt{\pi}}\right)}{2b}$	40

input
```
int(x*cosh(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sinh(b*x^2+a)/b
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x\cosh\left(a+bx^2\right)\,dx=\frac{\sinh\left(bx^2+a\right)}{2\,b}$$

input
```
integrate(x*cosh(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*sinh(b*x^2 + a)/b
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int x\cosh \left(a+bx^2\right)\,dx=\begin{cases}\frac{\sinh\left(a+bx^2\right)}{2b} & \text{for } b\neq 0\\ \frac{x^2\cosh\left(a\right)}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*cosh(b*x**2+a),x)
```

output
```
Piecewise((sinh(a + b*x**2)/(2*b), Ne(b, 0)), (x**2*cosh(a)/2, True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x\cosh \left(a+bx^2\right)\,dx=\frac{\sinh\left(bx^2+a\right)}{2\,b}$$

input
```
integrate(x*cosh(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*sinh(b*x^2 + a)/b
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int x\cosh \left(a+bx^2\right)\,dx=\frac{e^{\left(bx^2+a\right)}-e^{\left(-bx^2-a\right)}}{4\,b}$$

input
```
integrate(x*cosh(b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(e^(b*x^2 + a) - e^(-b*x^2 - a))/b
```

3.3.9 Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x \cosh \left(a+bx^2\right) \, dx = \frac{\sinh(b\,x^2+a)}{2\,b}$$

input
```
int(x*cosh(a + b*x^2),x)
```

output
```
sinh(a + b*x^2)/(2*b)
```

3.4 $\int \cosh(a+bx^2)\,dx$

3.4.1 Optimal result

Integrand size = 8, antiderivative size = 53

$$\int \cosh(a+bx^2)\,dx = \frac{e^{-a}\sqrt{\pi}\text{erf}\left(\sqrt{b}x\right)}{4\sqrt{b}} + \frac{e^{a}\sqrt{\pi}\text{erfi}\left(\sqrt{b}x\right)}{4\sqrt{b}}$$

output
```
1/4*erf(x*b^(1/2))*Pi^(1/2)/exp(a)/b^(1/2)+1/4*exp(a)*erfi(x*b^(1/2))*Pi^(
1/2)/b^(1/2)
```

3.4.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \cosh(a+bx^2)\,dx = \frac{\sqrt{\pi}\left(\text{erf}\left(\sqrt{b}x\right)(\cosh(a)-\sinh(a))+\text{erfi}\left(\sqrt{b}x\right)(\cosh(a)+\sinh(a))\right)}{4\sqrt{b}}$$

input
```
Integrate[Cosh[a + b*x^2],x]
```

output
```
(Sqrt[Pi]*(Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) + Erfi[Sqrt[b]*x]*(Cosh[a] +
 Sinh[a])))/(4*Sqrt[b])
```

3.4.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5822, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh\left(a + bx^2\right) \, dx$$

$$\Big\downarrow \text{ 5822}$$

$$\frac{1}{2} \int e^{-bx^2-a} dx + \frac{1}{2} \int e^{bx^2+a} dx$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{1}{2} \int e^{-bx^2-a} dx + \frac{\sqrt{\pi} e^a \text{erfi}\left(\sqrt{b} x\right)}{4\sqrt{b}}$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{\sqrt{\pi} e^{-a} \text{erf}\left(\sqrt{b} x\right)}{4\sqrt{b}} + \frac{\sqrt{\pi} e^a \text{erfi}\left(\sqrt{b} x\right)}{4\sqrt{b}}$$

input
```
Int[Cosh[a + b*x^2],x]
```

output
```
(Sqrt[Pi]*Erf[Sqrt[b]*x])/(4*Sqrt[b]*E^a) + (E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])
/(4*Sqrt[b])
```

3.4.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 5822
```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[1/2   Int[E^(c + d*x^n
), x], x] + Simp[1/2   Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IG
tQ[n, 1]
```

3.4.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{\operatorname{erf}\left(x\sqrt{b}\right)\sqrt{\pi}\,\mathrm{e}^{-a}}{4\sqrt{b}}+\frac{\mathrm{e}^{a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b}\,x\right)}{4\sqrt{-b}}$	40
meijerg	$\frac{\cosh(a)\sqrt{\pi}\,\sqrt{2}\left(\frac{\sqrt{ib}\,\sqrt{2}\,\operatorname{erf}\left(x\sqrt{b}\right)}{2\sqrt{b}}+\frac{\sqrt{ib}\,\sqrt{2}\,\operatorname{erfi}\left(x\sqrt{b}\right)}{2\sqrt{b}}\right)}{4\sqrt{ib}}-\frac{i\sinh(a)\sqrt{\pi}\,\sqrt{2}\left(-\frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\operatorname{erf}\left(x\sqrt{b}\right)}{2b^{\frac{3}{2}}}+\frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\operatorname{erfi}\left(x\sqrt{b}\right)}{2b^{\frac{3}{2}}}\right)}{4\sqrt{ib}}$	117

input
```
int(cosh(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*erf(x*b^(1/2))*Pi^(1/2)/exp(a)/b^(1/2)+1/4*exp(a)*Pi^(1/2)/(-b)^(1/2)*
erf((-b)^(1/2)*x)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \cosh\left(a+bx^2\right)\,dx = -\frac{\sqrt{\pi}\sqrt{-b}(\cosh(a)+\sinh(a))\operatorname{erf}\left(\sqrt{-b}x\right)-\sqrt{\pi}\sqrt{b}(\cosh(a)-\sinh(a))\operatorname{erf}\left(\sqrt{b}x\right)}{4\,b}$$

input
```
integrate(cosh(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)*x) - sqrt(pi)*sqr
t(b)*(cosh(a) - sinh(a))*erf(sqrt(b)*x))/b
```

3.4.6 Sympy [F]

$$\int \cosh(a+bx^2)\,dx = \int \cosh(a+bx^2)\,dx$$

input
```
integrate(cosh(b*x**2+a),x)
```

output
```
Integral(cosh(a + b*x**2), x)
```

3.4.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(35) = 70.

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.64

$$\int \cosh(a+bx^2)\,dx = -\frac{1}{4}b\left(\frac{2\,xe^{(bx^2+a)}}{b}+\frac{2\,xe^{(-bx^2-a)}}{b}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}x\right)e^{(-a)}}{b^{\frac{3}{2}}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-b}x\right)e^{a}}{\sqrt{-b}b}\right) + x\cosh\left(bx^2+a\right)$$

input
```
integrate(cosh(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/4*b*(2*x*e^(b*x^2 + a)/b + 2*x*e^(-b*x^2 - a)/b - sqrt(pi)*erf(sqrt(b)*
x)*e^(-a)/b^(3/2) - sqrt(pi)*erf(sqrt(-b)*x)*e^a/(sqrt(-b)*b)) + x*cosh(b*
x^2 + a)
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \cosh(a+bx^2)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{b}x\right)e^{(-a)}}{4\sqrt{b}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-b}x\right)e^{a}}{4\sqrt{-b}}$$

input
```
integrate(cosh(b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-sqrt(b)*x)*e^(-a)/sqrt(b) - 1/4*sqrt(pi)*erf(-sqrt(-b)*
x)*e^a/sqrt(-b)
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int \cosh\left(a + bx^2\right) \, dx = \int \cosh\left(b\,x^2 + a\right) \, dx$$

input
```
int(cosh(a + b*x^2),x)
```

output
```
int(cosh(a + b*x^2), x)
```

3.5 $\int \frac{\cosh(a+bx^2)}{x}\,dx$

3.5.1 Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{\cosh(a+bx^2)}{x}\,dx = \frac{1}{2}\cosh(a)\text{Chi}(bx^2) + \frac{1}{2}\sinh(a)\text{Shi}(bx^2)$$

output
```
1/2*Chi(b*x^2)*cosh(a)+1/2*Shi(b*x^2)*sinh(a)
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cosh(a+bx^2)}{x}\,dx = \frac{1}{2}\left(\cosh(a)\text{Chi}(bx^2) + \sinh(a)\text{Shi}(bx^2)\right)$$

input
```
Integrate[Cosh[a + b*x^2]/x,x]
```

output
```
(Cosh[a]*CoshIntegral[b*x^2] + Sinh[a]*SinhIntegral[b*x^2])/2
```

3.5.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5842, 5839, 5840}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx^2)}{x} dx$$

$$\Big\downarrow \text{5842}$$

$$\sinh(a)\int \frac{\sinh(bx^2)}{x} dx + \cosh(a)\int \frac{\cosh(bx^2)}{x} dx$$

$$\Big\downarrow \text{5839}$$

$$\cosh(a)\int \frac{\cosh(bx^2)}{x} dx + \frac{1}{2}\sinh(a)\text{Shi}(bx^2)$$

$$\Big\downarrow \text{5840}$$

$$\frac{1}{2}\cosh(a)\text{Chi}(bx^2) + \frac{1}{2}\sinh(a)\text{Shi}(bx^2)$$

input
```
Int[Cosh[a + b*x^2]/x,x]
```

output
```
(Cosh[a]*CoshIntegral[b*x^2])/2 + (Sinh[a]*SinhIntegral[b*x^2])/2
```

3.5.3.1 Defintions of rubi rules used

rule 5839
```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

rule 5840
```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

rule 5842
```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[Cosh[c]   Int[Cosh
[d*x^n]/x, x], x] + Simp[Sinh[c]   Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d
, n}, x]
```

3.5.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

method	result	size
risch	$-\frac{e^{2a} e^{-a} \operatorname{Ei}_1(-b x^2)}{4}-\frac{e^{-a} \operatorname{Ei}_1(b x^2)}{4}$	33
meijerg	$\frac{\cosh(a)\sqrt{\pi}\left(\frac{2\gamma+4\ln(x)+2\ln(ib)}{\sqrt{\pi}}+\frac{2\,\mathrm{Chi}(b x^2)-2\ln(b x^2)-2\gamma}{\sqrt{\pi}}\right)}{4}+\frac{\mathrm{Shi}(b x^2)\sinh(a)}{2}$	62

input
```
int(cosh(b*x^2+a)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(2*a)*exp(-a)*Ei(1,-b*x^2)-1/4*exp(-a)*Ei(1,b*x^2)
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{\cosh(a+bx^2)}{x}\,dx = \frac{1}{4}\left(\mathrm{Ei}(bx^2)+\mathrm{Ei}(-bx^2)\right)\cosh(a)+\frac{1}{4}\left(\mathrm{Ei}(bx^2)-\mathrm{Ei}(-bx^2)\right)\sinh(a)$$

input
```
integrate(cosh(b*x^2+a)/x,x, algorithm="fricas")
```

output
```
1/4*(Ei(b*x^2) + Ei(-b*x^2))*cosh(a) + 1/4*(Ei(b*x^2) - Ei(-b*x^2))*sinh(a
)
```

3.5.6 Sympy [F]

$$\int \frac{\cosh(a+bx^2)}{x}\,dx = \int \frac{\cosh(a+bx^2)}{x}\,dx$$

input
```
integrate(cosh(b*x**2+a)/x,x)
```

output
```
Integral(cosh(a + b*x**2)/x, x)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\cosh(a+bx^2)}{x}\,dx = \frac{1}{4}\,\mathrm{Ei}\left(-bx^2\right)e^{(-a)} + \frac{1}{4}\,\mathrm{Ei}\left(bx^2\right)e^{a}$$

input
```
integrate(cosh(b*x^2+a)/x,x, algorithm="maxima")
```

output
```
1/4*Ei(-b*x^2)*e^(-a) + 1/4*Ei(b*x^2)*e^a
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\cosh(a+bx^2)}{x}\,dx = \frac{1}{4}\,\mathrm{Ei}\left(-bx^2\right)e^{(-a)} + \frac{1}{4}\,\mathrm{Ei}\left(bx^2\right)e^{a}$$

input
```
integrate(cosh(b*x^2+a)/x,x, algorithm="giac")
```

output
```
1/4*Ei(-b*x^2)*e^(-a) + 1/4*Ei(b*x^2)*e^a
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx^2)}{x}\,dx = \frac{\cosh(a)\,\mathrm{coshint}(b\,x^2)}{2} + \frac{\sinh(a)\,\mathrm{sinhint}(b\,x^2)}{2}$$

input
```
int(cosh(a + b*x^2)/x,x)
```

output
```
(cosh(a)*coshint(b*x^2))/2 + (sinh(a)*sinhint(b*x^2))/2
```

3.6 $\int \frac{\cosh(a+bx^2)}{x^2} dx$

3.6.1 Optimal result

Integrand size = 12, antiderivative size = 66

$$\int \frac{\cosh(a+bx^2)}{x^2} dx = -\frac{\cosh(a+bx^2)}{x} - \frac{1}{2}\sqrt{b}e^{-a}\sqrt{\pi}\text{erf}\left(\sqrt{b}x\right) + \frac{1}{2}\sqrt{b}e^{a}\sqrt{\pi}\text{erfi}\left(\sqrt{b}x\right)$$

output
```
-cosh(b*x^2+a)/x-1/2*erf(x*b^(1/2))*b^(1/2)*Pi^(1/2)/exp(a)+1/2*exp(a)*erf
i(x*b^(1/2))*b^(1/2)*Pi^(1/2)
```

3.6.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(a+bx^2)}{x^2} dx = \frac{-2\cosh(a+bx^2) + \sqrt{b}\sqrt{\pi}x\text{erf}\left(\sqrt{b}x\right)(-\cosh(a)+\sinh(a)) + \sqrt{b}\sqrt{\pi}x\text{erfi}\left(\sqrt{b}x\right)(\cosh(a)+\sinh(a))}{2x}$$

input
```
Integrate[Cosh[a + b*x^2]/x^2,x]
```

output
```
(-2*Cosh[a + b*x^2] + Sqrt[b]*Sqrt[Pi]*x*Erf[Sqrt[b]*x]*(-Cosh[a] + Sinh[a
]) + Sqrt[b]*Sqrt[Pi]*x*Erfi[Sqrt[b]*x]*(Cosh[a] + Sinh[a]))/(2*x)
```

3.6.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5850, 5821, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx^2)}{x^2}\,dx$$

$$\Big\downarrow \text{5850}$$

$$2b\int \sinh(bx^2+a)\,dx-\frac{\cosh(a+bx^2)}{x}$$

$$\Big\downarrow \text{5821}$$

$$2b\left(\frac{1}{2}\int e^{bx^2+a}dx-\frac{1}{2}\int e^{-bx^2-a}dx\right)-\frac{\cosh(a+bx^2)}{x}$$

$$\Big\downarrow \text{2633}$$

$$2b\left(\frac{\sqrt{\pi}e^a\text{erfi}\left(\sqrt{b}x\right)}{4\sqrt{b}}-\frac{1}{2}\int e^{-bx^2-a}dx\right)-\frac{\cosh(a+bx^2)}{x}$$

$$\Big\downarrow \text{2634}$$

$$2b\left(\frac{\sqrt{\pi}e^a\text{erfi}\left(\sqrt{b}x\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}e^{-a}\text{erf}\left(\sqrt{b}x\right)}{4\sqrt{b}}\right)-\frac{\cosh(a+bx^2)}{x}$$

input
```
Int[Cosh[a + b*x^2]/x^2,x]
```

output
```
-(Cosh[a + b*x^2]/x) + 2*b*(-1/4*(Sqrt[Pi]*Erf[Sqrt[b]*x])/(Sqrt[b]*E^a) +
(E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/(4*Sqrt[b]))
```

3.6.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 5821
```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[1/2   Int[E^(c + d*x^n
), x], x] - Simp[1/2   Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IG
tQ[n, 1]
```

rule 5850
```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x
)^(m + 1)*(Cosh[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   In
t[(e*x)^(m + n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

3.6.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{e^{-a} e^{-b x^2}}{2x} - \frac{\operatorname{erf}\left(x\sqrt{b}\right)\sqrt{b}\sqrt{\pi}\, e^{-a}}{2} - \frac{e^{a} e^{b x^2}}{2x} + \frac{e^{a} b\sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-b}\, x\right)}{2\sqrt{-b}}$
meijerg	$\frac{i\cosh(a)\sqrt{\pi}\, b\sqrt{2}\left(-\frac{2\sqrt{2}\, e^{b x^2}}{\sqrt{\pi}\, x\sqrt{ib}} - \frac{2\sqrt{2}\, e^{-b x^2}}{\sqrt{\pi}\, x\sqrt{ib}} - \frac{2\sqrt{2}\sqrt{b}\, \operatorname{erf}\left(x\sqrt{b}\right)}{\sqrt{ib}} + \frac{2\sqrt{2}\sqrt{b}\, \operatorname{erfi}\left(x\sqrt{b}\right)}{\sqrt{ib}}\right)}{8\sqrt{ib}} + \frac{\sinh(a)\sqrt{\pi}\, b\sqrt{2}\left(\frac{2\sqrt{2}\sqrt{ib}\, e^{-b x^2}}{\sqrt{\pi}\, x b} - \frac{2\sqrt{2}\sqrt{ib}\, e^{b x^2}}{\sqrt{\pi}\, x b} \ldots\right)}{8\sqrt{ib}}$

input
```
int(cosh(b*x^2+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/exp(a)/x*exp(-b*x^2)-1/2*erf(x*b^(1/2))*b^(1/2)*Pi^(1/2)/exp(a)-1/2*e
xp(a)*exp(b*x^2)/x+1/2*exp(a)*b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(48) = 96.

Time = 0.25 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.77

$$\int \frac{\cosh(a+bx^2)}{x^2}\,dx =$$
$$-\frac{\sqrt{\pi}(x\cosh(bx^2+a)\cosh(a)+x\cosh(bx^2+a)\sinh(a)+(x\cosh(a)+x\sinh(a))\sinh(bx^2+a))\sqrt{-b}}{}$$

input
```
integrate(cosh(b*x^2+a)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(sqrt(pi)*(x*cosh(b*x^2 + a)*cosh(a) + x*cosh(b*x^2 + a)*sinh(a) + (x
*cosh(a) + x*sinh(a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(-b)*x) + sqrt(pi)
*(x*cosh(b*x^2 + a)*cosh(a) - x*cosh(b*x^2 + a)*sinh(a) + (x*cosh(a) - x*s
inh(a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(b)*x) + cosh(b*x^2 + a)^2 + 2*co
sh(b*x^2 + a)*sinh(b*x^2 + a) + sinh(b*x^2 + a)^2 + 1)/(x*cosh(b*x^2 + a)
+ x*sinh(b*x^2 + a))
```

3.6.6 Sympy [F]

$$\int \frac{\cosh(a+bx^2)}{x^2}\,dx = \int \frac{\cosh(a+bx^2)}{x^2}\,dx$$

input
```
integrate(cosh(b*x**2+a)/x**2,x)
```

output
```
Integral(cosh(a + b*x**2)/x**2, x)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \frac{\cosh(a+bx^2)}{x^2}\,dx = -\frac{1}{2}\left(\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}x\right)e^{(-a)}}{\sqrt{b}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-b}x\right)e^{a}}{\sqrt{-b}}\right)b - \frac{\cosh(bx^2+a)}{x}$$

input
```
integrate(cosh(b*x^2+a)/x^2,x, algorithm="maxima")
```

output
```
-1/2*(sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/sqrt(b) - sqrt(pi)*erf(sqrt(-b)*x)*e^
a/sqrt(-b))*b - cosh(b*x^2 + a)/x
```

3.6.8 Giac [F]

$$\int \frac{\cosh(a+bx^2)}{x^2}\,dx = \int \frac{\cosh(bx^2+a)}{x^2}\,dx$$

input
```
integrate(cosh(b*x^2+a)/x^2,x, algorithm="giac")
```

output
```
integrate(cosh(b*x^2 + a)/x^2, x)
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx^2)}{x^2}\,dx = \int \frac{\cosh(b\,x^2+a)}{x^2}\,dx$$

input
```
int(cosh(a + b*x^2)/x^2,x)
```

output
```
int(cosh(a + b*x^2)/x^2, x)
```

3.7 $\int \frac{\cosh(a+bx^2)}{x^3} dx$

3.7.1 Optimal result

Integrand size = 12, antiderivative size = 42

$$\int \frac{\cosh(a+bx^2)}{x^3} dx = -\frac{\cosh(a+bx^2)}{2x^2} + \frac{1}{2}b\text{Chi}(bx^2)\sinh(a) + \frac{1}{2}b\cosh(a)\text{Shi}(bx^2)$$

output
```
-1/2*cosh(b*x^2+a)/x^2+1/2*b*cosh(a)*Shi(b*x^2)+1/2*b*Chi(b*x^2)*sinh(a)
```

3.7.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{\cosh(a+bx^2)}{x^3} dx = \frac{1}{2}\left(-\frac{\cosh(a+bx^2)}{x^2} + b\text{Chi}(bx^2)\sinh(a) + b\cosh(a)\text{Shi}(bx^2)\right)$$

input
```
Integrate[Cosh[a + b*x^2]/x^3,x]
```

output
```
(-(Cosh[a + b*x^2]/x^2) + b*CoshIntegral[b*x^2]*Sinh[a] + b*Cosh[a]*SinhIn
tegral[b*x^2])/2
```

3.7.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5844, 3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx^2)}{x^3}dx$$

$$\Big\downarrow \text{5844}$$

$$\frac{1}{2}\int \frac{\cosh(bx^2+a)}{x^4}dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sin\left(ibx^2+ia+\frac{\pi}{2}\right)}{x^4}dx^2$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}\left(-\frac{\cosh(a+bx^2)}{x^2}+ib\int -\frac{i\sinh(bx^2+a)}{x^2}dx^2\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\left(b\int \frac{\sinh(bx^2+a)}{x^2}dx^2-\frac{\cosh(a+bx^2)}{x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-\frac{\cosh(a+bx^2)}{x^2}+b\int -\frac{i\sin(ibx^2+ia)}{x^2}dx^2\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\left(-\frac{\cosh(a+bx^2)}{x^2}-ib\int \frac{\sin(ibx^2+ia)}{x^2}dx^2\right)$$

$$\Big\downarrow \text{3784}$$

$$\frac{1}{2}\left(-\frac{\cosh(a+bx^2)}{x^2}-ib\left(i\sinh(a)\int \frac{\cosh(bx^2)}{x^2}dx^2+\cosh(a)\int \frac{i\sinh(bx^2)}{x^2}dx^2\right)\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\left(-\frac{\cosh\left(a+bx^2\right)}{x^2}-ib\left(i\sinh(a)\int\frac{\cosh\left(bx^2\right)}{x^2}dx^2+i\cosh(a)\int\frac{\sinh\left(bx^2\right)}{x^2}dx^2\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-\frac{\cosh\left(a+bx^2\right)}{x^2}-ib\left(i\sinh(a)\int\frac{\sin\left(ibx^2+\frac{\pi}{2}\right)}{x^2}dx^2+i\cosh(a)\int-\frac{i\sin\left(ibx^2\right)}{x^2}dx^2\right)\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\left(-\frac{\cosh\left(a+bx^2\right)}{x^2}-ib\left(i\sinh(a)\int\frac{\sin\left(ibx^2+\frac{\pi}{2}\right)}{x^2}dx^2+\cosh(a)\int\frac{\sin\left(ibx^2\right)}{x^2}dx^2\right)\right)$$

$$\Big\downarrow \text{3779}$$

$$\frac{1}{2}\left(-\frac{\cosh\left(a+bx^2\right)}{x^2}-ib\left(i\sinh(a)\int\frac{\sin\left(ibx^2+\frac{\pi}{2}\right)}{x^2}dx^2+i\cosh(a)\text{Shi}\left(bx^2\right)\right)\right)$$

$$\Big\downarrow \text{3782}$$

$$\frac{1}{2}\left(-\frac{\cosh\left(a+bx^2\right)}{x^2}-ib\left(i\sinh(a)\text{Chi}\left(bx^2\right)+i\cosh(a)\text{Shi}\left(bx^2\right)\right)\right)$$

input
```
Int[Cosh[a + b*x^2]/x^3,x]
```

output
```
(-(Cosh[a + b*x^2]/x^2) - I*b*(I*CoshIntegral[b*x^2]*Sinh[a] + I*Cosh[a]*S
inhIntegral[b*x^2]))/2
```

3.7.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

3.7.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

method	result
risch	$-\frac{-{\mathrm e}^{-a} \operatorname{Ei}_{1}\left(b \,x^{2}\right) b \,x^{2}+\operatorname{Ei}_{1}\left(-b \,x^{2}\right) {\mathrm e}^{a} b \,x^{2}+{\mathrm e}^{-b \,x^{2}-a}+{\mathrm e}^{b \,x^{2}+a}}{4 x^{2}}$
meijerg	$\frac{i \cosh \left(a \right) \sqrt{\pi}\, b \left(\frac{4 i \cosh \left(b \,x^{2}\right)}{b \,x^{2} \sqrt{\pi}}-\frac{4 i \,\operatorname{Shi}\left(b \,x^{2}\right)}{\sqrt{\pi}}\right)}{8}+\frac{\sinh \left(a \right) \sqrt{\pi}\, b \left(\frac{4 \gamma -4+8 \ln \left(x \right)+4 \ln \left(i b \right)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4 \sinh \left(b \,x^{2}\right)}{\sqrt{\pi}\, b \,x^{2}}+\frac{4 \,\operatorname{Chi}\left(b \,x^{2}\right)-4 \ln \left(b \,x^{2}\right)-4 \gamma}{\sqrt{\pi}}\right)}{8}$

input
```
int(cosh(b*x^2+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-exp(-a)*Ei(1,b*x^2)*b*x^2+Ei(1,-b*x^2)*exp(a)*b*x^2+exp(-b*x^2-a)+e
xp(b*x^2+a))/x^2
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

$$\int \frac{\cosh(a+bx^2)}{x^3}\,dx = \frac{(bx^2\mathrm{Ei}(bx^2) - bx^2\mathrm{Ei}(-bx^2))\cosh(a) + (bx^2\mathrm{Ei}(bx^2) + bx^2\mathrm{Ei}(-bx^2))\sinh(a) - 2\cosh(bx^2+a)}{4x^2}$$

input
```
integrate(cosh(b*x^2+a)/x^3,x, algorithm="fricas")
```

output
```
1/4*((b*x^2*Ei(b*x^2) - b*x^2*Ei(-b*x^2))*cosh(a) + (b*x^2*Ei(b*x^2) + b*x
^2*Ei(-b*x^2))*sinh(a) - 2*cosh(b*x^2 + a))/x^2
```

3.7.6 Sympy [F]

$$\int \frac{\cosh(a+bx^2)}{x^3}\,dx = \int \frac{\cosh(a+bx^2)}{x^3}\,dx$$

input
```
integrate(cosh(b*x**2+a)/x**3,x)
```

output
```
Integral(cosh(a + b*x**2)/x**3, x)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{\cosh(a+bx^2)}{x^3}\,dx = -\frac{1}{4}\left(\mathrm{Ei}(-bx^2)\,e^{(-a)} - \mathrm{Ei}(bx^2)\,e^{a}\right)b - \frac{\cosh(bx^2+a)}{2x^2}$$

input
```
integrate(cosh(b*x^2+a)/x^3,x, algorithm="maxima")
```

output
```
-1/4*(Ei(-b*x^2)*e^(-a) - Ei(b*x^2)*e^a)*b - 1/2*cosh(b*x^2 + a)/x^2
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(36) = 72.

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.57

$$\int \frac{\cosh(a+bx^2)}{x^3}\,dx = -\frac{(bx^2+a)b^2\mathrm{Ei}(-bx^2)\,e^{(-a)} - ab^2\mathrm{Ei}(-bx^2)\,e^{(-a)} - (bx^2+a)b^2\mathrm{Ei}(bx^2)\,e^{a} + ab^2\mathrm{Ei}(bx^2)\,e^{a} + b^2e^{(bx^2+a)} + b}{4\,b^2x^2}$$

input
```
integrate(cosh(b*x^2+a)/x^3,x, algorithm="giac")
```

output
```
-1/4*((b*x^2 + a)*b^2*Ei(-b*x^2)*e^(-a) - a*b^2*Ei(-b*x^2)*e^(-a) - (b*x^2
 + a)*b^2*Ei(b*x^2)*e^a + a*b^2*Ei(b*x^2)*e^a + b^2*e^(b*x^2 + a) + b^2*e^
(-b*x^2 - a))/(b^2*x^2)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx^2)}{x^3}\,dx = \int \frac{\cosh(b\,x^2+a)}{x^3}\,dx$$

input
```
int(cosh(a + b*x^2)/x^3,x)
```

output
```
int(cosh(a + b*x^2)/x^3, x)
```

3.8 $\int x^3 \cosh^2(a+bx^2)\, dx$

3.8.1 Optimal result

Integrand size = 14, antiderivative size = 51

$$\int x^3 \cosh^2(a+bx^2)\, dx = \frac{x^4}{8} - \frac{\cosh^2(a+bx^2)}{8b^2} + \frac{x^2\cosh(a+bx^2)\sinh(a+bx^2)}{4b}$$

output
```
1/8*x^4-1/8*cosh(b*x^2+a)^2/b^2+1/4*x^2*cosh(b*x^2+a)*sinh(b*x^2+a)/b
```

3.8.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^3 \cosh^2(a+bx^2)\, dx = -\frac{\cosh(2(a+bx^2)) - 2bx^2(bx^2+\sinh(2(a+bx^2)))}{16b^2}$$

input
```
Integrate[x^3*Cosh[a + b*x^2]^2,x]
```

output
```
-1/16*(Cosh[2*(a + b*x^2)] - 2*b*x^2*(b*x^2 + Sinh[2*(a + b*x^2)]))/b^2
```

3.8.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5844, 3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cosh^2\left(a+bx^2\right) \, dx$$

$$\Big\downarrow \text{5844}$$

$$\frac{1}{2} \int x^2 \cosh^2\left(bx^2+a\right) dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2} \int x^2 \sin\left(ibx^2+ia+\frac{\pi}{2}\right)^2 dx^2$$

$$\Big\downarrow \text{3791}$$

$$\frac{1}{2}\left(\frac{\int x^2 dx^2}{2}-\frac{\cosh^2\left(a+bx^2\right)}{4b^2}+\frac{x^2 \sinh\left(a+bx^2\right) \cosh\left(a+bx^2\right)}{2b}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\left(-\frac{\cosh^2\left(a+bx^2\right)}{4b^2}+\frac{x^2 \sinh\left(a+bx^2\right) \cosh\left(a+bx^2\right)}{2b}+\frac{x^4}{4}\right)$$

input
```
Int[x^3*Cosh[a + b*x^2]^2,x]
```

output
```
(x^4/4 - Cosh[a + b*x^2]^2/(4*b^2) + (x^2*Cosh[a + b*x^2]*Sinh[a + b*x^2])
/(2*b))/2
```

3.8.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

3.8.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$\frac{2b^2x^4+2b\,x^2\sinh\left(2b\,x^2+2a\right)-\cosh\left(2b\,x^2+2a\right)+1}{16b^2}$	46
risch	$\frac{x^4}{8}+\frac{\left(2b\,x^2-1\right)\mathrm{e}^{2b\,x^2+2a}}{32b^2}-\frac{\left(2b\,x^2+1\right)\mathrm{e}^{-2b\,x^2-2a}}{32b^2}$	55

input
```
int(x^3*cosh(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(2*b^2*x^4+2*b*x^2*sinh(2*b*x^2+2*a)-cosh(2*b*x^2+2*a)+1)/b^2
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int x^3\cosh^2\left(a+bx^2\right)\,dx = \frac{2\,b^2x^4+4\,bx^2\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)-\cosh\left(bx^2+a\right)^2-\sinh\left(bx^2+a\right)^2}{16\,b^2}$$

input
```
integrate(x^3*cosh(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/16*(2*b^2*x^4 + 4*b*x^2*cosh(b*x^2 + a)*sinh(b*x^2 + a) - cosh(b*x^2 + a
)^2 - sinh(b*x^2 + a)^2)/b^2
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int x^3\cosh^2\left(a+bx^2\right)\,dx = \begin{cases} -\frac{x^4\sinh^2\left(a+bx^2\right)}{8}+\frac{x^4\cosh^2\left(a+bx^2\right)}{8}+\frac{x^2\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b}-\frac{\cosh^2\left(a+bx^2\right)}{8b^2} & \text{for } b\neq 0\\ \frac{x^4\cosh^2\left(a\right)}{4} & \text{otherwise}\end{cases}$$

input
```
integrate(x**3*cosh(b*x**2+a)**2,x)
```

output
```
Piecewise((-x**4*sinh(a + b*x**2)**2/8 + x**4*cosh(a + b*x**2)**2/8 + x**2
*sinh(a + b*x**2)*cosh(a + b*x**2)/(4*b) - cosh(a + b*x**2)**2/(8*b**2), N
e(b, 0)), (x**4*cosh(a)**2/4, True))
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int x^3\cosh^2\left(a+bx^2\right)\,dx = \frac{1}{8}\,x^4+\frac{\left(2\,bx^2e^{(2\,a)}-e^{(2\,a)}\right)e^{\left(2\,bx^2\right)}}{32\,b^2}-\frac{\left(2\,bx^2+1\right)e^{\left(-2\,bx^2-2\,a\right)}}{32\,b^2}$$

input
```
integrate(x^3*cosh(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/8*x^4 + 1/32*(2*b*x^2*e^(2*a) - e^(2*a))*e^(2*b*x^2)/b^2 - 1/32*(2*b*x^2
 + 1)*e^(-2*b*x^2 - 2*a)/b^2
```

3.8.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(45) = 90$.

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.78

$$\int x^3 \cosh^2(a+bx^2)\,dx = \frac{4(bx^2+a)^2 + 2(bx^2+a)e^{(2bx^2+2a)} - 2(bx^2+a)e^{(-2bx^2-2a)} - e^{(2bx^2+2a)} - e^{(-2bx^2-2a)}}{32b^2} - \frac{4(bx^2+a)a + ae^{(2bx^2+2a)} - \left(2ae^{(2bx^2+2a)} + a\right)e^{(-2bx^2-2a)}}{16b^2}$$

input
```
integrate(x^3*cosh(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/32*(4*(b*x^2 + a)^2 + 2*(b*x^2 + a)*e^(2*b*x^2 + 2*a) - 2*(b*x^2 + a)*e^
(-2*b*x^2 - 2*a) - e^(2*b*x^2 + 2*a) - e^(-2*b*x^2 - 2*a))/b^2 - 1/16*(4*(
b*x^2 + a)*a + a*e^(2*b*x^2 + 2*a) - (2*a*e^(2*b*x^2 + 2*a) + a)*e^(-2*b*x
^2 - 2*a))/b^2
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int x^3 \cosh^2(a+bx^2)\,dx = \frac{x^4}{8} - \frac{\frac{\cosh(2bx^2+2a)}{16} - \frac{bx^2\sinh(2bx^2+2a)}{8}}{b^2}$$

input
```
int(x^3*cosh(a + b*x^2)^2,x)
```

output
```
x^4/8 - (cosh(2*a + 2*b*x^2)/16 - (b*x^2*sinh(2*a + 2*b*x^2))/8)/b^2
```

3.9 $\int x^2 \cosh^2(a+bx^2)\,dx$

3.9.1 Optimal result

Integrand size = 14, antiderivative size = 99

$$\int x^2 \cosh^2(a+bx^2)\,dx = \frac{x^3}{6} + \frac{e^{-2a}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{b}x\right)}{32b^{3/2}} - \frac{e^{2a}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{b}x\right)}{32b^{3/2}} + \frac{x\sinh(2a+2bx^2)}{8b}$$

output
```
1/6*x^3+1/8*x*sinh(2*b*x^2+2*a)/b+1/64*erf(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(
1/2)/b^(3/2)/exp(2*a)-1/64*exp(2*a)*erfi(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(1/
2)/b^(3/2)
```

3.9.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02

$$\int x^2 \cosh^2(a+bx^2)\,dx = \frac{3\sqrt{2\pi}\text{erf}\left(\sqrt{2}\sqrt{b}x\right)(\cosh(2a)-\sinh(2a)) - 3\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{b}x\right)(\cosh(2a)+\sinh(2a)) + 8\sqrt{b}x(4bx^2+3\,\text{s}}{192b^{3/2}}$$

input
```
Integrate[x^2*Cosh[a + b*x^2]^2,x]
```

output
```
(3*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] - Sinh[2*a]) - 3*Sqrt[2*Pi
]*Erfi[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] + Sinh[2*a]) + 8*Sqrt[b]*x*(4*b*x^2 +
 3*Sinh[2*(a + b*x^2)]))/(192*b^(3/2))
```

3.9.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5864, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cosh^2\left(a+bx^2\right) \, dx$$

$$\Big\downarrow \ 5864$$

$$\int \left(\frac{1}{2} x^2 \cosh\left(2a+2bx^2\right)+\frac{x^2}{2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{\frac{\pi}{2}} e^{-2a} \text{erf}\left(\sqrt{2}\sqrt{b}x\right)}{32b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}} e^{2a} \text{erfi}\left(\sqrt{2}\sqrt{b}x\right)}{32b^{3/2}}+\frac{x \sinh\left(2a+2bx^2\right)}{8b}+\frac{x^3}{6}$$

input
```
Int[x^2*Cosh[a + b*x^2]^2,x]
```

output
```
x^3/6 + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(32*b^(3/2)*E^(2*a)) - (E^(2*a
)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/(32*b^(3/2)) + (x*Sinh[2*a + 2*b*x^2
])/(8*b)
```

3.9.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5864
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

3.9.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{x^3}{6}-\frac{e^{-2a}x\,e^{-2b\,x^2}}{16b}+\frac{e^{-2a}\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(x\sqrt{2}\,\sqrt{b}\right)}{64b^{\frac{3}{2}}}+\frac{e^{2a}x\,e^{2b\,x^2}}{16b}-\frac{e^{2a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-2b}\,x\right)}{32b\sqrt{-2b}}$	90

input
```
int(x^2*cosh(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^3-1/16*exp(-2*a)/b*x*exp(-2*b*x^2)+1/64*exp(-2*a)/b^(3/2)*Pi^(1/2)*2
^(1/2)*erf(x*2^(1/2)*b^(1/2))+1/16*exp(2*a)/b*x*exp(2*b*x^2)-1/32*exp(2*a)
/b*Pi^(1/2)/(-2*b)^(1/2)*erf((-2*b)^(1/2)*x)
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(71) = 142$.

Time = 0.26 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.31

$$\int x^2\cosh^2\left(a+bx^2\right)\,dx$$
$$=\frac{32\,b^2x^3\cosh\left(bx^2+a\right)^2+12\,bx\cosh\left(bx^2+a\right)^4+48\,bx\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^3+12\,bx\sinh\left(bx^2+a\right.}{}$$

input
```
integrate(x^2*cosh(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/192*(32*b^2*x^3*cosh(b*x^2 + a)^2 + 12*b*x*cosh(b*x^2 + a)^4 + 48*b*x*co
sh(b*x^2 + a)*sinh(b*x^2 + a)^3 + 12*b*x*sinh(b*x^2 + a)^4 + 3*sqrt(2)*sqr
t(pi)*(cosh(b*x^2 + a)^2*cosh(2*a) + (cosh(2*a) + sinh(2*a))*sinh(b*x^2 +
a)^2 + cosh(b*x^2 + a)^2*sinh(2*a) + 2*(cosh(b*x^2 + a)*cosh(2*a) + cosh(b
*x^2 + a)*sinh(2*a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(2)*sqrt(-b)*x) + 3
*sqrt(2)*sqrt(pi)*(cosh(b*x^2 + a)^2*cosh(2*a) + (cosh(2*a) - sinh(2*a))*s
inh(b*x^2 + a)^2 - cosh(b*x^2 + a)^2*sinh(2*a) + 2*(cosh(b*x^2 + a)*cosh(2
*a) - cosh(b*x^2 + a)*sinh(2*a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(2)*sqrt
(b)*x) + 8*(4*b^2*x^3 + 9*b*x*cosh(b*x^2 + a)^2)*sinh(b*x^2 + a)^2 - 12*b*
x + 16*(4*b^2*x^3*cosh(b*x^2 + a) + 3*b*x*cosh(b*x^2 + a)^3)*sinh(b*x^2 +
a))/(b^2*cosh(b*x^2 + a)^2 + 2*b^2*cosh(b*x^2 + a)*sinh(b*x^2 + a) + b^2*s
inh(b*x^2 + a)^2)
```

3.9.6 Sympy [F]

$$\int x^2 \cosh^2(a + bx^2)\, dx = \int x^2 \cosh^2(a + bx^2)\, dx$$

input
```
integrate(x**2*cosh(b*x**2+a)**2,x)
```

output
```
Integral(x**2*cosh(a + b*x**2)**2, x)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int x^2 \cosh^2(a + bx^2)\, dx = \frac{1}{6}x^3 - \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{-b}x\right)e^{(2a)}}{64\sqrt{-b}b} + \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b}x\right)e^{(-2a)}}{64\,b^{\frac{3}{2}}} + \frac{xe^{(2bx^2+2a)}}{16\,b} - \frac{xe^{(-2bx^2-2a)}}{16\,b}$$

input
```
integrate(x^2*cosh(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/6*x^3 - 1/64*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(-b)*x)*e^(2*a)/(sqrt(-b)*
b) + 1/64*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(b)*x)*e^(-2*a)/b^(3/2) + 1/16*
x*e^(2*b*x^2 + 2*a)/b - 1/16*x*e^(-2*b*x^2 - 2*a)/b
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

$$\int x^2 \cosh^2(a + bx^2)\, dx = \frac{1}{6}x^3 + \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{2}\sqrt{-b}x\right)e^{(2a)}}{64\sqrt{-b}b} - \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{2}\sqrt{b}x\right)e^{(-2a)}}{64\,b^{\frac{3}{2}}} + \frac{xe^{(2bx^2+2a)}}{16\,b} - \frac{xe^{(-2bx^2-2a)}}{16\,b}$$

input
```
integrate(x^2*cosh(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/6*x^3 + 1/64*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(-b)*x)*e^(2*a)/(sqrt(-b)
*b) - 1/64*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(b)*x)*e^(-2*a)/b^(3/2) + 1/1
6*x*e^(2*b*x^2 + 2*a)/b - 1/16*x*e^(-2*b*x^2 - 2*a)/b
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh^2\left(a+bx^2\right)\,dx=\int x^2\,\cosh\left(b\,x^2+a\right)^2\,dx$$

input
```
int(x^2*cosh(a + b*x^2)^2,x)
```

output
```
int(x^2*cosh(a + b*x^2)^2, x)
```

3.10 $\int x \cosh^2(a+bx^2)\,dx$

3.10.1 Optimal result

Integrand size = 12, antiderivative size = 31

$$\int x \cosh^2(a+bx^2)\,dx = \frac{x^2}{4} + \frac{\cosh(a+bx^2)\sinh(a+bx^2)}{4b}$$

output
```
1/4*x^2+1/4*cosh(b*x^2+a)*sinh(b*x^2+a)/b
```

3.10.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int x \cosh^2(a+bx^2)\,dx = \frac{2(a+bx^2)+\sinh(2(a+bx^2))}{8b}$$

input
```
Integrate[x*Cosh[a + b*x^2]^2,x]
```

output
```
(2*(a + b*x^2) + Sinh[2*(a + b*x^2)])/(8*b)
```

3.10.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5844, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh^2\left(a+bx^2\right)\,dx$$

$$\Big\downarrow \text{5844}$$

$$\frac{1}{2}\int \cosh^2\left(bx^2+a\right)dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin\left(ibx^2+ia+\frac{\pi}{2}\right)^2dx^2$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{\int 1dx^2}{2}+\frac{\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{2b}\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left(\frac{\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{2b}+\frac{x^2}{2}\right)$$

input
```
Int[x*Cosh[a + b*x^2]^2,x]
```

output
```
(x^2/2 + (Cosh[a + b*x^2]*Sinh[a + b*x^2])/(2*b))/2
```

3.10.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

3.10.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
parallelrisch	$\frac{2 b \,x^{2}+\sinh \left(2 b \,x^{2}+2 a \right)}{8 b}$	24
derivativedivides	$\frac{\frac{\cosh \left(b \,x^{2}+a \right) \sinh \left(b \,x^{2}+a \right)}{2}+\frac{b \,x^{2}}{2}+\frac{a}{2}}{2 b}$	34
default	$\frac{\frac{\cosh \left(b \,x^{2}+a \right) \sinh \left(b \,x^{2}+a \right)}{2}+\frac{b \,x^{2}}{2}+\frac{a}{2}}{2 b}$	34
risch	$\frac{x^{2}}{4}+\frac{{\mathrm e}^{2 b \,x^{2}+2 a}}{16 b}-\frac{{\mathrm e}^{-2 b \,x^{2}-2 a}}{16 b}$	39

input
```
int(x*cosh(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*b*x^2+sinh(2*b*x^2+2*a))/b
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int x \cosh^2\left(a+bx^2\right)\,dx = \frac{bx^2+\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)}{4\,b}$$

input
```
integrate(x*cosh(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/4*(b*x^2 + cosh(b*x^2 + a)*sinh(b*x^2 + a))/b
```

3.10.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(24) = 48$.

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int x\cosh^2\left(a+bx^2\right)\,dx = \begin{cases} -\frac{x^2\sinh^2\left(a+bx^2\right)}{4}+\frac{x^2\cosh^2\left(a+bx^2\right)}{4}+\frac{\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b} & \text{for } b\neq 0 \\ \frac{x^2\cosh^2\left(a\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*cosh(b*x**2+a)**2,x)
```

output
```
Piecewise((-x**2*sinh(a + b*x**2)**2/4 + x**2*cosh(a + b*x**2)**2/4 + sinh
(a + b*x**2)*cosh(a + b*x**2)/(4*b), Ne(b, 0)), (x**2*cosh(a)**2/2, True))
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int x\cosh^2\left(a+bx^2\right)\,dx = \frac{1}{4}\,x^2+\frac{e^{\left(2\,bx^2+2\,a\right)}}{16\,b}-\frac{e^{\left(-2\,bx^2-2\,a\right)}}{16\,b}$$

input
```
integrate(x*cosh(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/4*x^2 + 1/16*e^(2*b*x^2 + 2*a)/b - 1/16*e^(-2*b*x^2 - 2*a)/b
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int x\cosh^2\left(a+bx^2\right)\,dx=\frac{4\,bx^2-\left(2\,e^{\left(2\,bx^2+2\,a\right)}+1\right)e^{\left(-2\,bx^2-2\,a\right)}+4\,a+e^{\left(2\,bx^2+2\,a\right)}}{16\,b}$$

input
```
integrate(x*cosh(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/16*(4*b*x^2 - (2*e^(2*b*x^2 + 2*a) + 1)*e^(-2*b*x^2 - 2*a) + 4*a + e^(2*
b*x^2 + 2*a))/b
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int x\cosh^2\left(a+bx^2\right)\,dx=\frac{\sinh(2\,b\,x^2+2\,a)}{8\,b}+\frac{x^2}{4}$$

input
```
int(x*cosh(a + b*x^2)^2,x)
```

output
```
sinh(2*a + 2*b*x^2)/(8*b) + x^2/4
```

3.11 $\int \cosh^2(a+bx^2)\,dx$

3.11.1 Optimal result

Integrand size = 10, antiderivative size = 78

$$\int \cosh^2(a+bx^2)\,dx=\frac{x}{2}+\frac{e^{-2a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{b}x\right)}{8\sqrt{b}}+\frac{e^{2a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{b}x\right)}{8\sqrt{b}}$$

output
```
1/2*x+1/16*erf(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(1/2)/exp(2*a)/b^(1/2)+1/16*e
xp(2*a)*erfi(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(1/2)/b^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int \cosh^2(a+bx^2)\,dx =\frac{4\sqrt{2}\sqrt{b}x+\sqrt{\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{b}x\right)(\cosh(2a)-\sinh(2a))+\sqrt{\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{b}x\right)(\cosh(2a)+\sinh(2a))}{8\sqrt{2}\sqrt{b}}$$

input
```
Integrate[Cosh[a + b*x^2]^2,x]
```

output
```
(4*Sqrt[2]*Sqrt[b]*x + Sqrt[Pi]*Erf[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] - Sinh[2
*a]) + Sqrt[Pi]*Erfi[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] + Sinh[2*a]))/(8*Sqrt[2
]*Sqrt[b])
```

3.11.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2\left(a+bx^2\right) dx$$

$$\Big\downarrow \text{5824}$$

$$\int \left(\frac{1}{2}\cosh\left(2a+2bx^2\right)+\frac{1}{2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\frac{\pi}{2}}e^{-2a}\text{erf}\left(\sqrt{2}\sqrt{b}x\right)}{8\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}e^{2a}\text{erfi}\left(\sqrt{2}\sqrt{b}x\right)}{8\sqrt{b}}+\frac{x}{2}$$

input
```
Int[Cosh[a + b*x^2]^2,x]
```

output
```
x/2 + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(8*Sqrt[b]*E^(2*a)) + (E^(2*a)*S
qrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/(8*Sqrt[b])
```

3.11.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5824
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[n, 1] && IGtQ[p, 1]
```

3.11.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{x}{2}+\frac{e^{-2a}\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(x\sqrt{2}\,\sqrt{b}\right)}{16\sqrt{b}}+\frac{e^{2a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-2b}\,x\right)}{8\sqrt{-2b}}$	51

input
```
int(cosh(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/16*exp(-2*a)*Pi^(1/2)*2^(1/2)/b^(1/2)*erf(x*2^(1/2)*b^(1/2))+1/8*e
xp(2*a)*Pi^(1/2)/(-2*b)^(1/2)*erf((-2*b)^(1/2)*x)
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \cosh^2\left(a+bx^2\right)\,dx =$$
$$-\frac{\sqrt{2}\sqrt{\pi}\sqrt{-b}(\cosh\left(2\,a\right)+\sinh\left(2\,a\right))\operatorname{erf}\left(\sqrt{2}\sqrt{-b}x\right)-\sqrt{2}\sqrt{\pi}\sqrt{b}(\cosh\left(2\,a\right)-\sinh\left(2\,a\right))\operatorname{erf}\left(\sqrt{2}\sqrt{b}x\right)-}{16\,b}$$

input
```
integrate(cosh(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(2)*sqrt(pi)*sqrt(-b)*(cosh(2*a) + sinh(2*a))*erf(sqrt(2)*sqrt(
-b)*x) - sqrt(2)*sqrt(pi)*sqrt(b)*(cosh(2*a) - sinh(2*a))*erf(sqrt(2)*sqrt
(b)*x) - 8*b*x)/b
```

3.11.6 Sympy [F]

$$\int \cosh^2\left(a+bx^2\right)\,dx=\int \cosh^2\left(a+bx^2\right)dx$$

input
```
integrate(cosh(b*x**2+a)**2,x)
```

output
```
Integral(cosh(a + b*x**2)**2, x)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.72

$$\int \cosh^2\left(a+bx^2\right)\,dx = \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{-b}x\right)e^{(2\,a)}}{16\,\sqrt{-b}} + \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b}x\right)e^{(-2\,a)}}{16\,\sqrt{b}} + \frac{1}{2}\,x$$

input
```
integrate(cosh(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(-b)*x)*e^(2*a)/sqrt(-b) + 1/16*sqrt
(2)*sqrt(pi)*erf(sqrt(2)*sqrt(b)*x)*e^(-2*a)/sqrt(b) + 1/2*x
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \cosh^2\left(a+bx^2\right)\,dx = -\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{2}\sqrt{-b}x\right)e^{(2\,a)}}{16\,\sqrt{-b}} - \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{2}\sqrt{b}x\right)e^{(-2\,a)}}{16\,\sqrt{b}} + \frac{1}{2}\,x$$

input
```
integrate(cosh(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(-b)*x)*e^(2*a)/sqrt(-b) - 1/16*sq
rt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(b)*x)*e^(-2*a)/sqrt(b) + 1/2*x
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \cosh^2\left(a+bx^2\right)\,dx = \int {\mathrm{cosh}\left(b\,x^2+a\right)}^2\,dx$$

input
```
int(cosh(a + b*x^2)^2,x)
```

output
```
int(cosh(a + b*x^2)^2, x)
```

3.12 $\int \frac{\cosh^2(a+bx^2)}{x} dx$

3.12.1 Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{\cosh^2(a+bx^2)}{x} dx = \frac{1}{4}\cosh(2a)\text{Chi}(2bx^2) + \frac{\log(x)}{2} + \frac{1}{4}\sinh(2a)\text{Shi}(2bx^2)$$

output
```
1/4*Chi(2*b*x^2)*cosh(2*a)+1/2*ln(x)+1/4*Shi(2*b*x^2)*sinh(2*a)
```

3.12.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^2(a+bx^2)}{x} dx = \frac{1}{4}\left(\cosh(2a)\text{Chi}(2bx^2) + 2\log(x) + \sinh(2a)\text{Shi}(2bx^2)\right)$$

input
```
Integrate[Cosh[a + b*x^2]^2/x,x]
```

output
```
(Cosh[2*a]*CoshIntegral[2*b*x^2] + 2*Log[x] + Sinh[2*a]*SinhIntegral[2*b*x
^2])/4
```

3.12.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5864, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2\left(a+bx^2\right)}{x} \, dx$$

$$\Big\downarrow \text{ 5864}$$

$$\int \left(\frac{\cosh\left(2a+2bx^2\right)}{2x}+\frac{1}{2x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\cosh(2a)\text{Chi}\left(2bx^2\right)+\frac{1}{4}\sinh(2a)\text{Shi}\left(2bx^2\right)+\frac{\log(x)}{2}$$

input
```
Int[Cosh[a + b*x^2]^2/x,x]
```

output
```
(Cosh[2*a]*CoshIntegral[2*b*x^2])/4 + Log[x]/2 + (Sinh[2*a]*SinhIntegral[2
*b*x^2])/4
```

3.12.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5864
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

3.12.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{\ln(x)}{2}-\frac{e^{-2a}\operatorname{Ei}_1(2bx^2)}{8}-\frac{e^{2a}\operatorname{Ei}_1(-2bx^2)}{8}$	34

input
```
int(cosh(b*x^2+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x)-1/8*exp(-2*a)*Ei(1,2*b*x^2)-1/8*exp(2*a)*Ei(1,-2*b*x^2)
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{\cosh^2(a+bx^2)}{x}\,dx = \frac{1}{8}\left(\operatorname{Ei}(2\,bx^2)+\operatorname{Ei}(-2\,bx^2)\right)\cosh(2\,a) + \frac{1}{8}\left(\operatorname{Ei}(2\,bx^2)-\operatorname{Ei}(-2\,bx^2)\right)\sinh(2\,a)+\frac{1}{2}\log(x)$$

input
```
integrate(cosh(b*x^2+a)^2/x,x, algorithm="fricas")
```

output
```
1/8*(Ei(2*b*x^2) + Ei(-2*b*x^2))*cosh(2*a) + 1/8*(Ei(2*b*x^2) - Ei(-2*b*x^
2))*sinh(2*a) + 1/2*log(x)
```

3.12.6 Sympy [F]

$$\int \frac{\cosh^2(a+bx^2)}{x}\,dx = \int \frac{\cosh^2(a+bx^2)}{x}\,dx$$

input
```
integrate(cosh(b*x**2+a)**2/x,x)
```

output
```
Integral(cosh(a + b*x**2)**2/x, x)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{\cosh^2(a+bx^2)}{x}\,dx = \frac{1}{8}\,\mathrm{Ei}\left(2\,bx^2\right)e^{(2\,a)} + \frac{1}{8}\,\mathrm{Ei}\left(-2\,bx^2\right)e^{(-2\,a)} + \frac{1}{2}\,\log\left(x\right)$$

input
```
integrate(cosh(b*x^2+a)^2/x,x, algorithm="maxima")
```

output
```
1/8*Ei(2*b*x^2)*e^(2*a) + 1/8*Ei(-2*b*x^2)*e^(-2*a) + 1/2*log(x)
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\cosh^2(a+bx^2)}{x}\,dx = \frac{1}{8}\,\mathrm{Ei}\left(2\,bx^2\right)e^{(2\,a)} + \frac{1}{8}\,\mathrm{Ei}\left(-2\,bx^2\right)e^{(-2\,a)} + \frac{1}{4}\,\log\left(bx^2\right)$$

input
```
integrate(cosh(b*x^2+a)^2/x,x, algorithm="giac")
```

output
```
1/8*Ei(2*b*x^2)*e^(2*a) + 1/8*Ei(-2*b*x^2)*e^(-2*a) + 1/4*log(b*x^2)
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx^2)}{x}\,dx = \int \frac{\cosh(b\,x^2+a)^2}{x}\,dx$$

input
```
int(cosh(a + b*x^2)^2/x,x)
```

output
```
int(cosh(a + b*x^2)^2/x, x)
```

3.13 $\int \frac{\cosh^2(a+bx^2)}{x^2} \, dx$

3.13.1 Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \frac{\cosh^2(a+bx^2)}{x^2} \, dx = -\frac{\cosh^2(a+bx^2)}{x} - \frac{1}{2}\sqrt{b}e^{-2a}\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{b}x\right) + \frac{1}{2}\sqrt{b}e^{2a}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{b}x\right)$$

output
```
-cosh(b*x^2+a)^2/x-1/4*erf(x*2^(1/2)*b^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/exp
(2*a)+1/4*exp(2*a)*erfi(x*2^(1/2)*b^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^2(a+bx^2)}{x^2} \, dx = \frac{-4\cosh^2(a+bx^2) + \sqrt{b}\sqrt{2\pi}x\text{erf}\left(\sqrt{2}\sqrt{b}x\right)(-\cosh(2a)+\sinh(2a)) + \sqrt{b}\sqrt{2\pi}x\text{erfi}\left(\sqrt{2}\sqrt{b}x\right)(\cosh(2a)}{4x}$$

input
```
Integrate[Cosh[a + b*x^2]^2/x^2,x]
```

output
```
(-4*Cosh[a + b*x^2]^2 + Sqrt[b]*Sqrt[2*Pi]*x*Erf[Sqrt[2]*Sqrt[b]*x]*(-Cosh
[2*a] + Sinh[2*a]) + Sqrt[b]*Sqrt[2*Pi]*x*Erfi[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*
a] + Sinh[2*a]))/(4*x)
```

3.13.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5854, 6151, 5837, 5821, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(a+bx^2)}{x^2} \, dx$$

$$\Big\downarrow \text{5854}$$

$$4b \int \cosh(bx^2+a)\sinh(bx^2+a)\,dx - \frac{\cosh^2(a+bx^2)}{x}$$

$$\Big\downarrow \text{6151}$$

$$2b \int \sinh(2(bx^2+a))\,dx - \frac{\cosh^2(a+bx^2)}{x}$$

$$\Big\downarrow \text{5837}$$

$$2b \int \sinh(2bx^2+2a)\,dx - \frac{\cosh^2(a+bx^2)}{x}$$

$$\Big\downarrow \text{5821}$$

$$2b\left(\frac{1}{2}\int e^{2bx^2+2a}dx - \frac{1}{2}\int e^{-2bx^2-2a}dx\right) - \frac{\cosh^2(a+bx^2)}{x}$$

$$\Big\downarrow \text{2633}$$

$$2b\left(\frac{\sqrt{\frac{\pi}{2}}e^{2a}\text{erfi}\left(\sqrt{2}\sqrt{b}x\right)}{4\sqrt{b}} - \frac{1}{2}\int e^{-2bx^2-2a}dx\right) - \frac{\cosh^2(a+bx^2)}{x}$$

$$\Big\downarrow \text{2634}$$

$$2b\left(\frac{\sqrt{\frac{\pi}{2}}e^{2a}\text{erfi}\left(\sqrt{2}\sqrt{b}x\right)}{4\sqrt{b}} - \frac{\sqrt{\frac{\pi}{2}}e^{-2a}\text{erf}\left(\sqrt{2}\sqrt{b}x\right)}{4\sqrt{b}}\right) - \frac{\cosh^2(a+bx^2)}{x}$$

input
```
Int[Cosh[a + b*x^2]^2/x^2,x]
```

output
```
-(Cosh[a + b*x^2]^2/x) + 2*b*(-1/4*(Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(Sq
rt[b]*E^(2*a)) + (E^(2*a)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/(4*Sqrt[b]))
```

3.13.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 5821
```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[1/2   Int[E^(c + d*x^n
), x], x] - Simp[1/2   Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IG
tQ[n, 1]
```

rule 5837
```
Int[((a_.) + (b_.)*Sinh[u_])^(p_.), x_Symbol] :> Int[(a + b*Sinh[ExpandToSu
m[u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] &&  !BinomialMatc
hQ[u, x]
```

rule 5854
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_)]^(p_)*(x_)^(m_.), x_Symbol] :> Simp[-Cosh[
a + b*x^n]^p/((n - 1)*x^(n - 1)), x] + Simp[b*n*(p/(n - 1))   Int[Cosh[a +
b*x^n]^(p - 1)*Sinh[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IntegersQ[n,
p] && EqQ[m + n, 0] && GtQ[p, 1] && NeQ[n, 1]
```

rule 6151
```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Simp[1/2^p   Int[u*Si
nh[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

3.13.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98

method	result	size
risch	$-\frac{1}{2x}-\frac{{\mathrm e}^{-2a}{\mathrm e}^{-2b\,x^{2}}}{4x}-\frac{{\mathrm e}^{-2a}\sqrt{b}\,\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(x\sqrt{2}\,\sqrt{b}\right)}{4}-\frac{{\mathrm e}^{2a}{\mathrm e}^{2b\,x^{2}}}{4x}+\frac{{\mathrm e}^{2a}b\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-2b}\,x\right)}{2\sqrt{-2b}}$	86

input
```
int(cosh(b*x^2+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x-1/4*exp(-2*a)/x*exp(-2*b*x^2)-1/4*exp(-2*a)*b^(1/2)*Pi^(1/2)*2^(1/2
)*erf(x*2^(1/2)*b^(1/2))-1/4*exp(2*a)/x*exp(2*b*x^2)+1/2*exp(2*a)*b*Pi^(1/
2)/(-2*b)^(1/2)*erf((-2*b)^(1/2)*x)
```

3.13.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(64) = 128$.

Time = 0.26 (sec) , antiderivative size = 394, normalized size of antiderivative = 4.48

$$\int \frac{\cosh^2(a+bx^2)}{x^2}\,dx =$$

$$-\frac{\cosh(bx^2+a)^4 + 4\cosh(bx^2+a)\sinh(bx^2+a)^3 + \sinh(bx^2+a)^4 + \sqrt{2}\sqrt{\pi}\Big(x\cosh(bx^2+a)^2\cosh(2}{}$$

input
```
integrate(cosh(b*x^2+a)^2/x^2,x, algorithm="fricas")
```

output
```
-1/4*(cosh(b*x^2 + a)^4 + 4*cosh(b*x^2 + a)*sinh(b*x^2 + a)^3 + sinh(b*x^2
 + a)^4 + sqrt(2)*sqrt(pi)*(x*cosh(b*x^2 + a)^2*cosh(2*a) + x*cosh(b*x^2 +
 a)^2*sinh(2*a) + (x*cosh(2*a) + x*sinh(2*a))*sinh(b*x^2 + a)^2 + 2*(x*cos
h(b*x^2 + a)*cosh(2*a) + x*cosh(b*x^2 + a)*sinh(2*a))*sinh(b*x^2 + a))*sqr
t(-b)*erf(sqrt(2)*sqrt(-b)*x) + sqrt(2)*sqrt(pi)*(x*cosh(b*x^2 + a)^2*cosh
(2*a) - x*cosh(b*x^2 + a)^2*sinh(2*a) + (x*cosh(2*a) - x*sinh(2*a))*sinh(b
*x^2 + a)^2 + 2*(x*cosh(b*x^2 + a)*cosh(2*a) - x*cosh(b*x^2 + a)*sinh(2*a)
)*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(2)*sqrt(b)*x) + 2*(3*cosh(b*x^2 + a)^2
 + 1)*sinh(b*x^2 + a)^2 + 2*cosh(b*x^2 + a)^2 + 4*(cosh(b*x^2 + a)^3 + cos
h(b*x^2 + a))*sinh(b*x^2 + a) + 1)/(x*cosh(b*x^2 + a)^2 + 2*x*cosh(b*x^2 +
 a)*sinh(b*x^2 + a) + x*sinh(b*x^2 + a)^2)
```

3.13.6 Sympy [F]

$$\int \frac{\cosh^2(a+bx^2)}{x^2}\,dx = \int \frac{\cosh^2(a+bx^2)}{x^2}\,dx$$

input
```
integrate(cosh(b*x**2+a)**2/x**2,x)
```

output
```
Integral(cosh(a + b*x**2)**2/x**2, x)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

$$\int \frac{\cosh^2(a+bx^2)}{x^2}\,dx = -\frac{\sqrt{2}\sqrt{bx^2}e^{(-2\,a)}\Gamma\left(-\frac{1}{2},2\,bx^2\right)}{8\,x} - \frac{\sqrt{2}\sqrt{-bx^2}e^{(2\,a)}\Gamma\left(-\frac{1}{2},-2\,bx^2\right)}{8\,x} - \frac{1}{2\,x}$$

input
```
integrate(cosh(b*x^2+a)^2/x^2,x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*sqrt(b*x^2)*e^(-2*a)*gamma(-1/2, 2*b*x^2)/x - 1/8*sqrt(2)*sqr
t(-b*x^2)*e^(2*a)*gamma(-1/2, -2*b*x^2)/x - 1/2/x
```

3.13.8 Giac [F]

$$\int \frac{\cosh^2(a+bx^2)}{x^2}\,dx = \int \frac{\cosh\left(bx^2+a\right)^2}{x^2}\,dx$$

input
```
integrate(cosh(b*x^2+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(cosh(b*x^2 + a)^2/x^2, x)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx^2)}{x^2}\,dx = \int \frac{\cosh(b\,x^2+a)^2}{x^2}\,dx$$

input
```
int(cosh(a + b*x^2)^2/x^2,x)
```

output
```
int(cosh(a + b*x^2)^2/x^2, x)
```

3.14 $\int \frac{\cosh^2(a+bx^2)}{x^3} dx$

3.14.1 Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{\cosh^2(a+bx^2)}{x^3} dx = -\frac{1}{4x^2} - \frac{\cosh(2(a+bx^2))}{4x^2} + \frac{1}{2}b\text{Chi}(2bx^2)\sinh(2a) + \frac{1}{2}b\cosh(2a)\text{Shi}(2bx^2)$$

output
```
-1/4/x^2-1/4*cosh(2*b*x^2+2*a)/x^2+1/2*b*cosh(2*a)*Shi(2*b*x^2)+1/2*b*Chi(
2*b*x^2)*sinh(2*a)
```

3.14.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{\cosh^2(a+bx^2)}{x^3} dx = \frac{1}{2}\left(-\frac{\cosh^2(a+bx^2)}{x^2} + b\text{Chi}(2bx^2)\sinh(2a) + b\cosh(2a)\text{Shi}(2bx^2)\right)$$

input
```
Integrate[Cosh[a + b*x^2]^2/x^3,x]
```

output
```
(-(Cosh[a + b*x^2]^2/x^2) + b*CoshIntegral[2*b*x^2]*Sinh[2*a] + b*Cosh[2*a
]*SinhIntegral[2*b*x^2])/2
```

3.14.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5864, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2\left(a+bx^2\right)}{x^3}dx$$

$$\Big\downarrow \; 5864$$

$$\int \left(\frac{\cosh\left(2a+2bx^2\right)}{2x^3}+\frac{1}{2x^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}b\sinh(2a)\text{Chi}\left(2bx^2\right)+\frac{1}{2}b\cosh(2a)\text{Shi}\left(2bx^2\right)-\frac{\cosh\left(2\left(a+bx^2\right)\right)}{4x^2}-\frac{1}{4x^2}$$

input
```
Int[Cosh[a + b*x^2]^2/x^3,x]
```

output
```
-1/4*1/x^2 - Cosh[2*(a + b*x^2)]/(4*x^2) + (b*CoshIntegral[2*b*x^2]*Sinh[2
*a])/2 + (b*Cosh[2*a]*SinhIntegral[2*b*x^2])/2
```

3.14.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5864
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

3.14.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

method	result	size
risch	$-\frac{-2\,\mathrm{e}^{-2a}\,\mathrm{Ei}_{1}\left(2b\,x^{2}\right)b\,x^{2}+2\,\mathrm{e}^{2a}\,\mathrm{Ei}_{1}\left(-2b\,x^{2}\right)b\,x^{2}+\mathrm{e}^{-2b\,x^{2}-2a}+\mathrm{e}^{2b\,x^{2}+2a}+2}{8x^{2}}$	66

input
```
int(cosh(b*x^2+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-2*exp(-2*a)*Ei(1,2*b*x^2)*b*x^2+2*exp(2*a)*Ei(1,-2*b*x^2)*b*x^2+exp
(-2*b*x^2-2*a)+exp(2*b*x^2+2*a)+2)/x^2
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{\cosh^2\left(a+bx^2\right)}{x^3}\,dx = -\frac{\cosh\left(bx^2+a\right)^2-\left(bx^2\mathrm{Ei}(2\,bx^2)-bx^2\mathrm{Ei}(-2\,bx^2)\right)\cosh\left(2\,a\right)+\sinh\left(bx^2+a\right)^2-\left(bx^2\mathrm{Ei}(2\,bx^2)+bx^2\mathrm{Ei}(\right.}{4\,x^2}$$

input
```
integrate(cosh(b*x^2+a)^2/x^3,x, algorithm="fricas")
```

output
```
-1/4*(cosh(b*x^2 + a)^2 - (b*x^2*Ei(2*b*x^2) - b*x^2*Ei(-2*b*x^2))*cosh(2*
a) + sinh(b*x^2 + a)^2 - (b*x^2*Ei(2*b*x^2) + b*x^2*Ei(-2*b*x^2))*sinh(2*a
) + 1)/x^2
```

3.14.6 Sympy [F]

$$\int \frac{\cosh^2\left(a+bx^2\right)}{x^3}\,dx = \int \frac{\cosh^2\left(a+bx^2\right)}{x^3}\,dx$$

input
```
integrate(cosh(b*x**2+a)**2/x**3,x)
```

output
```
Integral(cosh(a + b*x**2)**2/x**3, x)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

$$\int \frac{\cosh^2(a+bx^2)}{x^3}\,dx = -\frac{1}{4}\,be^{(-2\,a)}\Gamma\left(-1,2\,bx^2\right) + \frac{1}{4}\,be^{(2\,a)}\Gamma\left(-1,-2\,bx^2\right) - \frac{1}{4\,x^2}$$

input
```
integrate(cosh(b*x^2+a)^2/x^3,x, algorithm="maxima")
```

output
```
-1/4*b*e^(-2*a)*gamma(-1, 2*b*x^2) + 1/4*b*e^(2*a)*gamma(-1, -2*b*x^2) - 1
/4/x^2
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(50) = 100.

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.21

$$\int \frac{\cosh^2(a+bx^2)}{x^3}\,dx$$
$$= \frac{2\,(bx^2+a)b^2\mathrm{Ei}(2\,bx^2)\,e^{(2\,a)} - 2\,ab^2\mathrm{Ei}(2\,bx^2)\,e^{(2\,a)} - 2\,(bx^2+a)b^2\mathrm{Ei}(-2\,bx^2)\,e^{(-2\,a)} + 2\,ab^2\mathrm{Ei}(-2\,bx^2)\,e^{(-2\,a}}{8\,b^2x^2}$$

input
```
integrate(cosh(b*x^2+a)^2/x^3,x, algorithm="giac")
```

output
```
1/8*(2*(b*x^2 + a)*b^2*Ei(2*b*x^2)*e^(2*a) - 2*a*b^2*Ei(2*b*x^2)*e^(2*a) -
 2*(b*x^2 + a)*b^2*Ei(-2*b*x^2)*e^(-2*a) + 2*a*b^2*Ei(-2*b*x^2)*e^(-2*a) -
 b^2*e^(2*b*x^2 + 2*a) - b^2*e^(-2*b*x^2 - 2*a) - 2*b^2)/(b^2*x^2)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx^2)}{x^3}\,dx = \int \frac{\cosh(b\,x^2+a)^2}{x^3}\,dx$$

input
```
int(cosh(a + b*x^2)^2/x^3,x)
```

output
```
int(cosh(a + b*x^2)^2/x^3, x)
```

3.15 $\int x^3 \cosh^3(a+bx^2)\,dx$

3.15.1 Optimal result

Integrand size = 14, antiderivative size = 79

$$\int x^3 \cosh^3\left(a+bx^2\right)\,dx = -\frac{\cosh(a+bx^2)}{3b^2} - \frac{\cosh^3(a+bx^2)}{18b^2} + \frac{x^2\sinh(a+bx^2)}{3b} + \frac{x^2\cosh^2(a+bx^2)\sinh(a+bx^2)}{6b}$$

output
```
-1/3*cosh(b*x^2+a)/b^2-1/18*cosh(b*x^2+a)^3/b^2+1/3*x^2*sinh(b*x^2+a)/b+1/
6*x^2*cosh(b*x^2+a)^2*sinh(b*x^2+a)/b
```

3.15.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

$$\int x^3 \cosh^3\left(a+bx^2\right)\,dx = -\frac{27\cosh(a+bx^2)+\cosh(3(a+bx^2))-3bx^2(9\sinh(a+bx^2)+\sinh(3(a+bx^2)))}{72b^2}$$

input
```
Integrate[x^3*Cosh[a + b*x^2]^3,x]
```

output
```
-1/72*(27*Cosh[a + b*x^2] + Cosh[3*(a + b*x^2)] - 3*b*x^2*(9*Sinh[a + b*x^
2] + Sinh[3*(a + b*x^2)]))/b^2
```

3.15.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5844, 3042, 3791, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cosh^3\left(a+bx^2\right) dx$$

$$\Big\downarrow \text{ 5844}$$

$$\frac{1}{2}\int x^2 \cosh^3\left(bx^2+a\right) dx^2$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int x^2 \sin\left(ibx^2+ia+\frac{\pi}{2}\right)^3 dx^2$$

$$\Big\downarrow \text{ 3791}$$

$$\frac{1}{2}\left(\frac{2}{3}\int x^2 \cosh\left(bx^2+a\right) dx^2-\frac{\cosh^3\left(a+bx^2\right)}{9b^2}+\frac{x^2\sinh\left(a+bx^2\right)\cosh^2\left(a+bx^2\right)}{3b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(\frac{2}{3}\int x^2 \sin\left(ibx^2+ia+\frac{\pi}{2}\right) dx^2-\frac{\cosh^3\left(a+bx^2\right)}{9b^2}+\frac{x^2\sinh\left(a+bx^2\right)\cosh^2\left(a+bx^2\right)}{3b}\right)$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{1}{2}\left(\frac{2}{3}\left(\frac{x^2\sinh\left(a+bx^2\right)}{b}-\frac{i\int -i\sinh\left(bx^2+a\right)dx^2}{b}\right)-\frac{\cosh^3\left(a+bx^2\right)}{9b^2}+\frac{x^2\sinh\left(a+bx^2\right)\cosh^2\left(a+bx^2\right)}{3b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{2}\left(\frac{2}{3}\left(\frac{x^2\sinh\left(a+bx^2\right)}{b}-\frac{\int \sinh\left(bx^2+a\right)dx^2}{b}\right)-\frac{\cosh^3\left(a+bx^2\right)}{9b^2}+\frac{x^2\sinh\left(a+bx^2\right)\cosh^2\left(a+bx^2\right)}{3b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(\frac{2}{3}\left(\frac{x^2\sinh\left(a+bx^2\right)}{b}-\frac{\int -i\sin\left(ibx^2+ia\right)dx^2}{b}\right)-\frac{\cosh^3\left(a+bx^2\right)}{9b^2}+\frac{x^2\sinh\left(a+bx^2\right)\cosh^2\left(a+bx^2\right)}{3b}\right)$$

$$\Big\downarrow \; 26$$

$$\frac{1}{2}\left(\frac{2}{3}\left(\frac{x^2\sinh\left(a+bx^2\right)}{b}+\frac{i\int\sin\left(ibx^2+ia\right)dx^2}{b}\right)-\frac{\cosh^3\left(a+bx^2\right)}{9b^2}+\frac{x^2\sinh\left(a+bx^2\right)\cosh^2\left(a+bx^2\right)}{3b}\right)$$

$$\Big\downarrow \; 3118$$

$$\frac{1}{2}\left(-\frac{\cosh^3\left(a+bx^2\right)}{9b^2}+\frac{2}{3}\left(\frac{x^2\sinh\left(a+bx^2\right)}{b}-\frac{\cosh\left(a+bx^2\right)}{b^2}\right)+\frac{x^2\sinh\left(a+bx^2\right)\cosh^2\left(a+bx^2\right)}{3b}\right)$$

input
```
Int[x^3*Cosh[a + b*x^2]^3,x]
```

output
```
(-1/9*Cosh[a + b*x^2]^3/b^2 + (x^2*Cosh[a + b*x^2]^2*Sinh[a + b*x^2])/(3*b
) + (2*(-(Cosh[a + b*x^2]/b^2) + (x^2*Sinh[a + b*x^2])/b))/3)/2
```

3.15.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

3.15.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.18

method	result	size
risch	$\frac{(3b x^2-1)e^{3b x^2+3a}}{144b^2}+\frac{3(b x^2-1)e^{b x^2+a}}{16b^2}-\frac{3(b x^2+1)e^{-b x^2-a}}{16b^2}-\frac{(3b x^2+1)e^{-3b x^2-3a}}{144b^2}$	93
parallelrisch	$\frac{-9\tanh\left(\frac{b x^2}{2}+\frac{a}{2}\right)^5 x^2 b+6\tanh\left(\frac{b x^2}{2}+\frac{a}{2}\right)^3 x^2 b+9\tanh\left(\frac{b x^2}{2}+\frac{a}{2}\right)^4-9\tanh\left(\frac{b x^2}{2}+\frac{a}{2}\right)x^2 b-12\tanh\left(\frac{b x^2}{2}+\frac{a}{2}\right)^2+7}{9b^2\left(1+\tanh\left(\frac{b x^2}{2}+\frac{a}{2}\right)\right)^3\left(\tanh\left(\frac{b x^2}{2}+\frac{a}{2}\right)-1\right)^3}$	123

input
```
int(x^3*cosh(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/144*(3*b*x^2-1)/b^2*exp(3*b*x^2+3*a)+3/16*(b*x^2-1)/b^2*exp(b*x^2+a)-3/1
6*(b*x^2+1)/b^2*exp(-b*x^2-a)-1/144*(3*b*x^2+1)/b^2*exp(-3*b*x^2-3*a)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.22

$$\int x^3\cosh^3\left(a+bx^2\right)\,dx$$
$$=\frac{3\,bx^2\sinh\left(bx^2+a\right)^3-\cosh\left(bx^2+a\right)^3-3\,\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^2+9\left(bx^2\cosh\left(bx^2+a\right)^2+3\,bx^2\right.}{72\,b^2}$$

input
```
integrate(x^3*cosh(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/72*(3*b*x^2*sinh(b*x^2 + a)^3 - cosh(b*x^2 + a)^3 - 3*cosh(b*x^2 + a)*si
nh(b*x^2 + a)^2 + 9*(b*x^2*cosh(b*x^2 + a)^2 + 3*b*x^2)*sinh(b*x^2 + a) -
27*cosh(b*x^2 + a))/b^2
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int x^3 \cosh^3\left(a+bx^2\right)\,dx$$
$$=\begin{cases} -\frac{x^2\sinh^3\left(a+bx^2\right)}{3b}+\frac{x^2\sinh\left(a+bx^2\right)\cosh^2\left(a+bx^2\right)}{2b}+\frac{\sinh^2\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{3b^2}-\frac{7\cosh^3\left(a+bx^2\right)}{18b^2} & \text{for } b\neq 0\\ \frac{x^4\cosh^3(a)}{4} & \text{otherwise}\end{cases}$$

input
```
integrate(x**3*cosh(b*x**2+a)**3,x)
```

output
```
Piecewise((-x**2*sinh(a + b*x**2)**3/(3*b) + x**2*sinh(a + b*x**2)*cosh(a
+ b*x**2)**2/(2*b) + sinh(a + b*x**2)**2*cosh(a + b*x**2)/(3*b**2) - 7*cos
h(a + b*x**2)**3/(18*b**2), Ne(b, 0)), (x**4*cosh(a)**3/4, True))
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.27

$$\int x^3\cosh^3\left(a+bx^2\right)\,dx=\frac{\left(3\,bx^2e^{(3\,a)}-e^{(3\,a)}\right)e^{\left(3\,bx^2\right)}}{144\,b^2}+\frac{3\left(bx^2e^a-e^a\right)e^{\left(bx^2\right)}}{16\,b^2}-\frac{3\left(bx^2+1\right)e^{\left(-bx^2-a\right)}}{16\,b^2}-\frac{\left(3\,bx^2+1\right)e^{\left(-3\,bx^2-3\,a\right)}}{144\,b^2}$$

input
```
integrate(x^3*cosh(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/144*(3*b*x^2*e^(3*a) - e^(3*a))*e^(3*b*x^2)/b^2 + 3/16*(b*x^2*e^a - e^a)
*e^(b*x^2)/b^2 - 3/16*(b*x^2 + 1)*e^(-b*x^2 - a)/b^2 - 1/144*(3*b*x^2 + 1)
*e^(-3*b*x^2 - 3*a)/b^2
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(71) = 142$.

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.43

$$\int x^3\cosh^3\left(a+bx^2\right)\,dx = \frac{3\,(bx^2+a)e^{(3\,bx^2+3\,a)}+27\,(bx^2+a)e^{(bx^2+a)}-27\,(bx^2+a)e^{(-bx^2-a)}-3\,(bx^2+a)e^{(-3\,bx^2-3\,a)}-e^{(3\,bx^2+3\,a)}}{144\,b^2} - \frac{ae^{(3\,bx^2+3\,a)}+9\,ae^{(bx^2+a)}-\left(9\,ae^{(2\,bx^2+2\,a)}+a\right)e^{(-3\,bx^2-3\,a)}}{48\,b^2}$$

input
```
integrate(x^3*cosh(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/144*(3*(b*x^2 + a)*e^(3*b*x^2 + 3*a) + 27*(b*x^2 + a)*e^(b*x^2 + a) - 27
*(b*x^2 + a)*e^(-b*x^2 - a) - 3*(b*x^2 + a)*e^(-3*b*x^2 - 3*a) - e^(3*b*x^
2 + 3*a) - 27*e^(b*x^2 + a) - 27*e^(-b*x^2 - a) - e^(-3*b*x^2 - 3*a))/b^2
- 1/48*(a*e^(3*b*x^2 + 3*a) + 9*a*e^(b*x^2 + a) - (9*a*e^(2*b*x^2 + 2*a) +
 a)*e^(-3*b*x^2 - 3*a))/b^2
```

3.15.9 Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int x^3\cosh^3\left(a+bx^2\right)\,dx = \frac{\frac{x^2\sinh\left(b\,x^2+a\right)}{3}+\frac{x^2\cosh\left(b\,x^2+a\right)^2\sinh\left(b\,x^2+a\right)}{6}}{b} - \frac{\cosh\left(b\,x^2+a\right)^3}{18\,b^2} - \frac{\cosh\left(b\,x^2+a\right)}{3\,b^2}$$

input
```
int(x^3*cosh(a + b*x^2)^3,x)
```

output
```
((x^2*sinh(a + b*x^2))/3 + (x^2*cosh(a + b*x^2)^2*sinh(a + b*x^2))/6)/b -
cosh(a + b*x^2)^3/(18*b^2) - cosh(a + b*x^2)/(3*b^2)
```

3.16 $\int x^2 \cosh^3\left(a+bx^2\right)\,dx$

3.16.1 Optimal result

Integrand size = 14, antiderivative size = 160

$$\int x^2 \cosh^3\left(a+bx^2\right)\,dx=\frac{3e^{-a}\sqrt{\pi}\text{erf}\left(\sqrt{b}x\right)}{32b^{3/2}}+\frac{e^{-3a}\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{b}x\right)}{96b^{3/2}}-\frac{3e^{a}\sqrt{\pi}\text{erfi}\left(\sqrt{b}x\right)}{32b^{3/2}}-\frac{e^{3a}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\sqrt{3}\sqrt{b}x\right)}{96b^{3/2}}+\frac{3x\sinh\left(a+bx^2\right)}{8b}+\frac{x\sinh\left(3a+3bx^2\right)}{24b}$$

output
```
3/8*x*sinh(b*x^2+a)/b+1/24*x*sinh(3*b*x^2+3*a)/b+1/288*erf(x*3^(1/2)*b^(1/
2))*3^(1/2)*Pi^(1/2)/b^(3/2)/exp(3*a)-1/288*exp(3*a)*erfi(x*3^(1/2)*b^(1/2
))*3^(1/2)*Pi^(1/2)/b^(3/2)+3/32*erf(x*b^(1/2))*Pi^(1/2)/b^(3/2)/exp(a)-3/
32*exp(a)*erfi(x*b^(1/2))*Pi^(1/2)/b^(3/2)
```

3.16.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.15

$$\int x^2 \cosh^3\left(a+bx^2\right)\,dx$$

$$=\frac{-27\sqrt{\pi}\cosh(a)\text{erfi}\left(\sqrt{b}x\right)-\sqrt{3\pi}\cosh(3a)\text{erfi}\left(\sqrt{3}\sqrt{b}x\right)+27\sqrt{\pi}\text{erf}\left(\sqrt{b}x\right)\left(\cosh(a)-\sinh(a)\right)-27\sqrt{\pi}\text{e}}{}$$

input
```
Integrate[x^2*Cosh[a + b*x^2]^3,x]
```

output
```
(-27*Sqrt[Pi]*Cosh[a]*Erfi[Sqrt[b]*x] - Sqrt[3*Pi]*Cosh[3*a]*Erfi[Sqrt[3]*
Sqrt[b]*x] + 27*Sqrt[Pi]*Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) - 27*Sqrt[Pi]*
Erfi[Sqrt[b]*x]*Sinh[a] + Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[b]*x]*(Cosh[3*a] - S
inh[3*a]) - Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[b]*x]*Sinh[3*a] + 108*Sqrt[b]*x*S
inh[a + b*x^2] + 12*Sqrt[b]*x*Sinh[3*(a + b*x^2)])/(288*b^(3/2))
```

3.16.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5864, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cosh^3\left(a+bx^2\right) dx$$

$$\Big\downarrow \text{5864}$$

$$\int \left(\frac{3}{4}x^2\cosh\left(a+bx^2\right)+\frac{1}{4}x^2\cosh\left(3a+3bx^2\right)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\pi}e^{-a}\text{erf}\left(\sqrt{b}x\right)}{32b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}e^{-3a}\text{erf}\left(\sqrt{3}\sqrt{b}x\right)}{96b^{3/2}}-\frac{3\sqrt{\pi}e^{a}\text{erfi}\left(\sqrt{b}x\right)}{32b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}e^{3a}\text{erfi}\left(\sqrt{3}\sqrt{b}x\right)}{96b^{3/2}}+\frac{3x\sinh\left(a+bx^2\right)}{8b}+\frac{x\sinh\left(3a+3bx^2\right)}{24b}$$

input
```
Int[x^2*Cosh[a + b*x^2]^3,x]
```

output
```
(3*Sqrt[Pi]*Erf[Sqrt[b]*x])/(32*b^(3/2)*E^a) + (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqr
t[b]*x])/(96*b^(3/2)*E^(3*a)) - (3*E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/(32*b^(3/
2)) - (E^(3*a)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[b]*x])/(96*b^(3/2)) + (3*x*Sin
h[a + b*x^2])/(8*b) + (x*Sinh[3*a + 3*b*x^2])/(24*b)
```

3.16.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5864
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

3.16.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{e^{-3a}x\,e^{-3b\,x^2}}{48b}+\frac{e^{-3a}\sqrt{\pi}\,\sqrt{3}\,\mathrm{erf}\left(x\sqrt{3}\,\sqrt{b}\right)}{288b^{\frac{3}{2}}}-\frac{3\,e^{-a}x\,e^{-b\,x^2}}{16b}+\frac{3\,\mathrm{erf}\left(x\sqrt{b}\right)\sqrt{\pi}\,e^{-a}}{32b^{\frac{3}{2}}}+\frac{e^{3a}x\,e^{3b\,x^2}}{48b}-\frac{e^{3a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-3b}\,x\right)}{96b\sqrt{-3b}}$

input
```
int(x^2*cosh(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/48/exp(a)^3/b*x*exp(-3*b*x^2)+1/288/exp(a)^3/b^(3/2)*Pi^(1/2)*3^(1/2)*e
rf(x*3^(1/2)*b^(1/2))-3/16/exp(a)/b*x*exp(-b*x^2)+3/32*erf(x*b^(1/2))*Pi^(
1/2)/b^(3/2)/exp(a)+1/48*exp(a)^3/b*x*exp(3*b*x^2)-1/96*exp(a)^3/b*Pi^(1/2
)/(-3*b)^(1/2)*erf((-3*b)^(1/2)*x)+3/16*exp(a)*exp(b*x^2)*x/b-3/32*exp(a)/
b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 903 vs. 2(114) = 228.

Time = 0.26 (sec) , antiderivative size = 903, normalized size of antiderivative = 5.64

$$\int x^2\cosh^3\left(a+bx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*cosh(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/288*(6*b*x*cosh(b*x^2 + a)^6 + 36*b*x*cosh(b*x^2 + a)*sinh(b*x^2 + a)^5
+ 6*b*x*sinh(b*x^2 + a)^6 + 54*b*x*cosh(b*x^2 + a)^4 + 18*(5*b*x*cosh(b*x^
2 + a)^2 + 3*b*x)*sinh(b*x^2 + a)^4 - 54*b*x*cosh(b*x^2 + a)^2 + 24*(5*b*x
*cosh(b*x^2 + a)^3 + 9*b*x*cosh(b*x^2 + a))*sinh(b*x^2 + a)^3 + sqrt(3)*sq
rt(pi)*(cosh(b*x^2 + a)^3*cosh(3*a) + (cosh(3*a) + sinh(3*a))*sinh(b*x^2 +
 a)^3 + cosh(b*x^2 + a)^3*sinh(3*a) + 3*(cosh(b*x^2 + a)*cosh(3*a) + cosh(
b*x^2 + a)*sinh(3*a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(3*a) +
 cosh(b*x^2 + a)^2*sinh(3*a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(3)*sqrt(-
b)*x) + sqrt(3)*sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(3*a) + (cosh(3*a) - sinh(
3*a))*sinh(b*x^2 + a)^3 - cosh(b*x^2 + a)^3*sinh(3*a) + 3*(cosh(b*x^2 + a)
*cosh(3*a) - cosh(b*x^2 + a)*sinh(3*a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2
+ a)^2*cosh(3*a) - cosh(b*x^2 + a)^2*sinh(3*a))*sinh(b*x^2 + a))*sqrt(b)*e
rf(sqrt(3)*sqrt(b)*x) + 27*sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(a) + (cosh(a)
+ sinh(a))*sinh(b*x^2 + a)^3 + cosh(b*x^2 + a)^3*sinh(a) + 3*(cosh(b*x^2 +
 a)*cosh(a) + cosh(b*x^2 + a)*sinh(a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 +
 a)^2*cosh(a) + cosh(b*x^2 + a)^2*sinh(a))*sinh(b*x^2 + a))*sqrt(-b)*erf(s
qrt(-b)*x) + 27*sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(a) + (cosh(a) - sinh(a))*
sinh(b*x^2 + a)^3 - cosh(b*x^2 + a)^3*sinh(a) + 3*(cosh(b*x^2 + a)*cosh(a)
 - cosh(b*x^2 + a)*sinh(a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(
a) - cosh(b*x^2 + a)^2*sinh(a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(b)*x)...
```

3.16.6 Sympy [F]

$$\int x^2 \cosh^3 \left(a+bx^2\right) \, dx = \int x^2 \cosh^3 \left(a+bx^2\right) dx$$

input
```
integrate(x**2*cosh(b*x**2+a)**3,x)
```

output
```
Integral(x**2*cosh(a + b*x**2)**3, x)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01

$$\int x^2\cosh^3\left(a+bx^2\right)\,dx = -\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{-b}x\right)e^{(3\,a)}}{288\,\sqrt{-b}b}+\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{b}x\right)e^{(-3\,a)}}{288\,b^{\frac{3}{2}}}+\frac{xe^{(3\,bx^2+3\,a)}}{48\,b}+\frac{3\,xe^{(bx^2+a)}}{16\,b}-\frac{3\,xe^{(-bx^2-a)}}{16\,b}-\frac{xe^{(-3\,bx^2-3\,a)}}{48\,b}+\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}x\right)e^{(-a)}}{32\,b^{\frac{3}{2}}}-\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-b}x\right)e^{a}}{32\,\sqrt{-b}b}$$

input
```
integrate(x^2*cosh(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/288*sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(-b)*x)*e^(3*a)/(sqrt(-b)*b) + 1/2
88*sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(b)*x)*e^(-3*a)/b^(3/2) + 1/48*x*e^(3*
b*x^2 + 3*a)/b + 3/16*x*e^(b*x^2 + a)/b - 3/16*x*e^(-b*x^2 - a)/b - 1/48*x
*e^(-3*b*x^2 - 3*a)/b + 3/32*sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/b^(3/2) - 3/32
*sqrt(pi)*erf(sqrt(-b)*x)*e^a/(sqrt(-b)*b)
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.04

$$\int x^2\cosh^3\left(a+bx^2\right)\,dx = \frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{3}\sqrt{-b}x\right)e^{(3\,a)}}{288\,\sqrt{-b}b}-\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{3}\sqrt{b}x\right)e^{(-3\,a)}}{288\,b^{\frac{3}{2}}}+\frac{xe^{(3\,bx^2+3\,a)}}{48\,b}+\frac{3\,xe^{(bx^2+a)}}{16\,b}-\frac{3\,xe^{(-bx^2-a)}}{16\,b}-\frac{xe^{(-3\,bx^2-3\,a)}}{48\,b}-\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{b}x\right)e^{(-a)}}{32\,b^{\frac{3}{2}}}+\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-b}x\right)e^{a}}{32\,\sqrt{-b}b}$$

input
```
integrate(x^2*cosh(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/288*sqrt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(-b)*x)*e^(3*a)/(sqrt(-b)*b) - 1/2
88*sqrt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(b)*x)*e^(-3*a)/b^(3/2) + 1/48*x*e^(3
*b*x^2 + 3*a)/b + 3/16*x*e^(b*x^2 + a)/b - 3/16*x*e^(-b*x^2 - a)/b - 1/48*
x*e^(-3*b*x^2 - 3*a)/b - 3/32*sqrt(pi)*erf(-sqrt(b)*x)*e^(-a)/b^(3/2) + 3/
32*sqrt(pi)*erf(-sqrt(-b)*x)*e^a/(sqrt(-b)*b)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh^3\left(a+bx^2\right)\,dx=\int x^2\cosh\left(b\,x^2+a\right)^3\,dx$$

input
```
int(x^2*cosh(a + b*x^2)^3,x)
```

output
```
int(x^2*cosh(a + b*x^2)^3, x)
```

3.17 $\int x\cosh^3\left(a+bx^2\right)\,dx$

3.17.1 Optimal result

Integrand size = 12, antiderivative size = 33

$$\int x\cosh^3\left(a+bx^2\right)\,dx=\frac{\sinh\left(a+bx^2\right)}{2b}+\frac{\sinh^3\left(a+bx^2\right)}{6b}$$

output
```
1/2*sinh(b*x^2+a)/b+1/6*sinh(b*x^2+a)^3/b
```

3.17.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x\cosh^3\left(a+bx^2\right)\,dx=\frac{\sinh\left(a+bx^2\right)}{2b}+\frac{\sinh^3\left(a+bx^2\right)}{6b}$$

input
```
Integrate[x*Cosh[a + b*x^2]^3,x]
```

output
```
Sinh[a + b*x^2]/(2*b) + Sinh[a + b*x^2]^3/(6*b)
```

3.17.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5844, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh^3\left(a+bx^2\right)\,dx$$

$$\Big\downarrow \text{5844}$$

$$\frac{1}{2}\int \cosh^3\left(bx^2+a\right)dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin\left(ibx^2+ia+\frac{\pi}{2}\right)^3dx^2$$

$$\Big\downarrow \text{3113}$$

$$\frac{i\int\left(1-x^4\right)d\left(-i\sinh\left(bx^2+a\right)\right)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{x^6}{3}-i\sinh\left(a+bx^2\right)\right)}{2b}$$

input
```
Int[x*Cosh[a + b*x^2]^3,x]
```

output
```
((I/2)*(-1/3*x^6 - I*Sinh[a + b*x^2]))/b
```

3.17.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

3.17.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\left(\frac{2}{3}+\frac{\cosh\left(b\,x^2+a\right)^2}{3}\right)\sinh\left(b\,x^2+a\right)}{2b}$	28
default	$\frac{\left(\frac{2}{3}+\frac{\cosh\left(b\,x^2+a\right)^2}{3}\right)\sinh\left(b\,x^2+a\right)}{2b}$	28
parallelrisch	$\frac{\sinh\left(3b\,x^2+3a\right)+9\sinh\left(b\,x^2+a\right)}{24b}$	28
risch	$\frac{\mathrm{e}^{3b\,x^2+3a}}{48b}+\frac{3\,\mathrm{e}^{b\,x^2+a}}{16b}-\frac{3\,\mathrm{e}^{-b\,x^2-a}}{16b}-\frac{\mathrm{e}^{-3b\,x^2-3a}}{48b}$	63

input
```
int(x*cosh(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(2/3+1/3*cosh(b*x^2+a)^2)*sinh(b*x^2+a)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int x\cosh^3\left(a+bx^2\right)\,dx=\frac{\sinh\left(bx^2+a\right)^3+3\left(\cosh\left(bx^2+a\right)^2+3\right)\sinh\left(bx^2+a\right)}{24\,b}$$

input
```
integrate(x*cosh(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/24*(sinh(b*x^2 + a)^3 + 3*(cosh(b*x^2 + a)^2 + 3)*sinh(b*x^2 + a))/b
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int x\cosh^3\left(a+bx^2\right)\,dx=\begin{cases}-\frac{\sinh^3\left(a+bx^2\right)}{3b}+\frac{\sinh\left(a+bx^2\right)\cosh^2\left(a+bx^2\right)}{2b} & \text{for } b\neq 0\\ \frac{x^2\cosh^3(a)}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*cosh(b*x**2+a)**3,x)
```

output
```
Piecewise((-sinh(a + b*x**2)**3/(3*b) + sinh(a + b*x**2)*cosh(a + b*x**2)*
*2/(2*b), Ne(b, 0)), (x**2*cosh(a)**3/2, True))
```

3.17.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

$$\int x\cosh^3\left(a+bx^2\right)\,dx=\frac{e^{\left(3\,bx^2+3\,a\right)}}{48\,b}+\frac{3\,e^{\left(bx^2+a\right)}}{16\,b}-\frac{3\,e^{\left(-bx^2-a\right)}}{16\,b}-\frac{e^{\left(-3\,bx^2-3\,a\right)}}{48\,b}$$

input
```
integrate(x*cosh(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/48*e^(3*b*x^2 + 3*a)/b + 3/16*e^(b*x^2 + a)/b - 3/16*e^(-b*x^2 - a)/b -
1/48*e^(-3*b*x^2 - 3*a)/b
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int x\cosh^3\left(a+bx^2\right)\,dx=-\frac{\left(9\,e^{\left(2\,bx^2+2\,a\right)}+1\right)e^{\left(-3\,bx^2-3\,a\right)}-e^{\left(3\,bx^2+3\,a\right)}-9\,e^{\left(bx^2+a\right)}}{48\,b}$$

input
```
integrate(x*cosh(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/48*((9*e^(2*b*x^2 + 2*a) + 1)*e^(-3*b*x^2 - 3*a) - e^(3*b*x^2 + 3*a) -
9*e^(b*x^2 + a))/b
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int x\cosh^3\left(a+bx^2\right)\,dx=\frac{\sinh(b\,x^2+a)^3+3\sinh(b\,x^2+a)}{6\,b}$$

input
```
int(x*cosh(a + b*x^2)^3,x)
```

output
```
(3*sinh(a + b*x^2) + sinh(a + b*x^2)^3)/(6*b)
```

3.18 $\int \cosh^3(a+bx^2)\,dx$

3.18.1 Optimal result

Integrand size = 10, antiderivative size = 125

$$\int \cosh^3(a+bx^2)\,dx = \frac{3e^{-a}\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}x\right)}{16\sqrt{b}} + \frac{e^{-3a}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}} + \frac{3e^{a}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}x\right)}{16\sqrt{b}} + \frac{e^{3a}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}}$$

output
```
1/48*erf(x*3^(1/2)*b^(1/2))*3^(1/2)*Pi^(1/2)/exp(3*a)/b^(1/2)+1/48*exp(3*a
)*erfi(x*3^(1/2)*b^(1/2))*3^(1/2)*Pi^(1/2)/b^(1/2)+3/16*erf(x*b^(1/2))*Pi^
(1/2)/exp(a)/b^(1/2)+3/16*exp(a)*erfi(x*b^(1/2))*Pi^(1/2)/b^(1/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.09

$$\int \cosh^3(a+bx^2)\,dx = \frac{\sqrt{\frac{\pi}{3}}\left(3\sqrt{3}\cosh(a)\operatorname{erfi}\left(\sqrt{b}x\right)+\cosh(3a)\operatorname{erfi}\left(\sqrt{3}\sqrt{b}x\right)+3\sqrt{3}\operatorname{erf}\left(\sqrt{b}x\right)(\cosh(a)-\sinh(a))+3\sqrt{3}\operatorname{erfi}\left(\sqrt{\right.\right.}{16\sqrt{b}}$$

input
```
Integrate[Cosh[a + b*x^2]^3,x]
```

output
```
(Sqrt[Pi/3]*(3*Sqrt[3]*Cosh[a]*Erfi[Sqrt[b]*x] + Cosh[3*a]*Erfi[Sqrt[3]*Sq
rt[b]*x] + 3*Sqrt[3]*Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) + 3*Sqrt[3]*Erfi[S
qrt[b]*x]*Sinh[a] + Erf[Sqrt[3]*Sqrt[b]*x]*(Cosh[3*a] - Sinh[3*a]) + Erfi[
Sqrt[3]*Sqrt[b]*x]*Sinh[3*a]))/(16*Sqrt[b])
```

3.18.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5824, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3\left(a+bx^2\right)\,dx$$

$$\Big\downarrow \text{5824}$$

$$\int \left(\frac{3}{4}\cosh\left(a+bx^2\right)+\frac{1}{4}\cosh\left(3a+3bx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\pi}e^{-a}\text{erf}\left(\sqrt{b}x\right)}{16\sqrt{b}}+\frac{\sqrt{\frac{\pi}{3}}e^{-3a}\text{erf}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}}+\frac{3\sqrt{\pi}e^{a}\text{erfi}\left(\sqrt{b}x\right)}{16\sqrt{b}}+\frac{\sqrt{\frac{\pi}{3}}e^{3a}\text{erfi}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}}$$

input
```
Int[Cosh[a + b*x^2]^3,x]
```

output
```
(3*Sqrt[Pi]*Erf[Sqrt[b]*x])/(16*Sqrt[b]*E^a) + (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqr
t[b]*x])/(16*Sqrt[b]*E^(3*a)) + (3*E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/(16*Sqrt[
b]) + (E^(3*a)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[b]*x])/(16*Sqrt[b])
```

3.18.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5824
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[n, 1] && IGtQ[p, 1]
```

3.18.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{e^{-3a}\sqrt{\pi}\,\sqrt{3}\,\operatorname{erf}\left(x\sqrt{3}\,\sqrt{b}\right)}{48\sqrt{b}}+\frac{3\,\operatorname{erf}\left(x\sqrt{b}\right)\sqrt{\pi}\,e^{-a}}{16\sqrt{b}}+\frac{e^{3a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-3b}\,x\right)}{16\sqrt{-3b}}+\frac{3\,e^{a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b}\,x\right)}{16\sqrt{-b}}$	86

input
```
int(cosh(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/48/exp(a)^3*Pi^(1/2)*3^(1/2)/b^(1/2)*erf(x*3^(1/2)*b^(1/2))+3/16*erf(x*b
^(1/2))*Pi^(1/2)/exp(a)/b^(1/2)+1/16*exp(a)^3*Pi^(1/2)/(-3*b)^(1/2)*erf((-
3*b)^(1/2)*x)+3/16*exp(a)*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\int \cosh^3\left(a+bx^2\right)\,dx =$$

$$-\frac{\sqrt{3}\sqrt{\pi}\sqrt{-b}(\cosh(3a)+\sinh(3a))\operatorname{erf}\left(\sqrt{3}\sqrt{-b}x\right)-\sqrt{3}\sqrt{\pi}\sqrt{b}(\cosh(3a)-\sinh(3a))\operatorname{erf}\left(\sqrt{3}\sqrt{b}x\right)+}{48\,b}$$

input
```
integrate(cosh(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/48*(sqrt(3)*sqrt(pi)*sqrt(-b)*(cosh(3*a) + sinh(3*a))*erf(sqrt(3)*sqrt(
-b)*x) - sqrt(3)*sqrt(pi)*sqrt(b)*(cosh(3*a) - sinh(3*a))*erf(sqrt(3)*sqrt
(b)*x) + 9*sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)*x) - 9*sqrt(
pi)*sqrt(b)*(cosh(a) - sinh(a))*erf(sqrt(b)*x))/b
```

3.18.6 Sympy [F]

$$\int \cosh^3\left(a+bx^2\right) dx = \int \cosh^3\left(a+bx^2\right) dx$$

input
```
integrate(cosh(b*x**2+a)**3,x)
```

output
```
Integral(cosh(a + b*x**2)**3, x)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int \cosh^3\left(a+bx^2\right) dx = \frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{-b}x\right)e^{(3a)}}{48\sqrt{-b}} + \frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{b}x\right)e^{(-3a)}}{48\sqrt{b}} + \frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}x\right)e^{(-a)}}{16\sqrt{b}} + \frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-b}x\right)e^{a}}{16\sqrt{-b}}$$

input
```
integrate(cosh(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/48*sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(-b)*x)*e^(3*a)/sqrt(-b) + 1/48*sqrt
(3)*sqrt(pi)*erf(sqrt(3)*sqrt(b)*x)*e^(-3*a)/sqrt(b) + 3/16*sqrt(pi)*erf(s
qrt(b)*x)*e^(-a)/sqrt(b) + 3/16*sqrt(pi)*erf(sqrt(-b)*x)*e^a/sqrt(-b)
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.76

$$\int \cosh^3\left(a+bx^2\right) dx = -\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{3}\sqrt{-b}x\right)e^{(3a)}}{48\sqrt{-b}} - \frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{3}\sqrt{b}x\right)e^{(-3a)}}{48\sqrt{b}} - \frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{b}x\right)e^{(-a)}}{16\sqrt{b}} - \frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-b}x\right)e^{a}}{16\sqrt{-b}}$$

input
```
integrate(cosh(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/48*sqrt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(-b)*x)*e^(3*a)/sqrt(-b) - 1/48*sq
rt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(b)*x)*e^(-3*a)/sqrt(b) - 3/16*sqrt(pi)*er
f(-sqrt(b)*x)*e^(-a)/sqrt(b) - 3/16*sqrt(pi)*erf(-sqrt(-b)*x)*e^a/sqrt(-b)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \cosh^3\left(a+bx^2\right)\,dx=\int \cosh\left(b\,x^2+a\right)^3\,dx$$

input
```
int(cosh(a + b*x^2)^3,x)
```

output
```
int(cosh(a + b*x^2)^3, x)
```

3.19 $\int \frac{\cosh^3(a+bx^2)}{x} dx$

3.19.1 Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{\cosh^3(a+bx^2)}{x} dx = \frac{3}{8}\cosh(a)\text{Chi}(bx^2) + \frac{1}{8}\cosh(3a)\text{Chi}(3bx^2) + \frac{3}{8}\sinh(a)\text{Shi}(bx^2) + \frac{1}{8}\sinh(3a)\text{Shi}(3bx^2)$$

output
```
3/8*Chi(b*x^2)*cosh(a)+1/8*Chi(3*b*x^2)*cosh(3*a)+3/8*Shi(b*x^2)*sinh(a)+1
/8*Shi(3*b*x^2)*sinh(3*a)
```

3.19.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^3(a+bx^2)}{x} dx = \frac{1}{8}\left(3\cosh(a)\text{Chi}(bx^2) + \cosh(3a)\text{Chi}(3bx^2) + 3\sinh(a)\text{Shi}(bx^2) + \sinh(3a)\text{Shi}(3bx^2)\right)$$

input
```
Integrate[Cosh[a + b*x^2]^3/x,x]
```

output
```
(3*Cosh[a]*CoshIntegral[b*x^2] + Cosh[3*a]*CoshIntegral[3*b*x^2] + 3*Sinh[
a]*SinhIntegral[b*x^2] + Sinh[3*a]*SinhIntegral[3*b*x^2])/8
```

3.19.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5864, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3\left(a+bx^2\right)}{x} \, dx$$

$$\Big\downarrow \text{ 5864}$$

$$\int \left(\frac{3\cosh\left(a+bx^2\right)}{4x}+\frac{\cosh\left(3a+3bx^2\right)}{4x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{8}\cosh(a)\text{Chi}\left(bx^2\right)+\frac{1}{8}\cosh(3a)\text{Chi}\left(3bx^2\right)+\frac{3}{8}\sinh(a)\text{Shi}\left(bx^2\right)+\frac{1}{8}\sinh(3a)\text{Shi}\left(3bx^2\right)$$

input
```
Int[Cosh[a + b*x^2]^3/x,x]
```

output
```
(3*Cosh[a]*CoshIntegral[b*x^2])/8 + (Cosh[3*a]*CoshIntegral[3*b*x^2])/8 +
(3*Sinh[a]*SinhIntegral[b*x^2])/8 + (Sinh[3*a]*SinhIntegral[3*b*x^2])/8
```

3.19.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5864
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

3.19.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

method	result	size
risch	$-\frac{\mathrm{e}^{6a}\mathrm{e}^{-3a}\,\mathrm{Ei}_1(-3bx^2)}{16}-\frac{3\,\mathrm{e}^{4a}\mathrm{e}^{-3a}\,\mathrm{Ei}_1(-bx^2)}{16}-\frac{3\,\mathrm{e}^{2a}\mathrm{e}^{-3a}\,\mathrm{Ei}_1(bx^2)}{16}-\frac{\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx^2)}{16}$	69

input
```
int(cosh(b*x^2+a)^3/x,x,method=_RETURNVERBOSE)
```

output
```
-1/16*exp(6*a)*exp(-3*a)*Ei(1,-3*b*x^2)-3/16*exp(4*a)*exp(-3*a)*Ei(1,-b*x^
2)-3/16*exp(2*a)*exp(-3*a)*Ei(1,b*x^2)-1/16*exp(-3*a)*Ei(1,3*b*x^2)
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{\cosh^3(a+bx^2)}{x}\,dx = \frac{1}{16}\left(\mathrm{Ei}(3\,bx^2)+\mathrm{Ei}(-3\,bx^2)\right)\cosh(3\,a) + \frac{3}{16}\left(\mathrm{Ei}(bx^2)+\mathrm{Ei}(-bx^2)\right)\cosh(a) + \frac{1}{16}\left(\mathrm{Ei}(3\,bx^2)-\mathrm{Ei}(-3\,bx^2)\right)\sinh(3\,a) + \frac{3}{16}\left(\mathrm{Ei}(bx^2)-\mathrm{Ei}(-bx^2)\right)\sinh(a)$$

input
```
integrate(cosh(b*x^2+a)^3/x,x, algorithm="fricas")
```

output
```
1/16*(Ei(3*b*x^2) + Ei(-3*b*x^2))*cosh(3*a) + 3/16*(Ei(b*x^2) + Ei(-b*x^2)
)*cosh(a) + 1/16*(Ei(3*b*x^2) - Ei(-3*b*x^2))*sinh(3*a) + 3/16*(Ei(b*x^2)
- Ei(-b*x^2))*sinh(a)
```

3.19.6 Sympy [F]

$$\int \frac{\cosh^3(a+bx^2)}{x}\,dx = \int \frac{\cosh^3(a+bx^2)}{x}\,dx$$

input
```
integrate(cosh(b*x**2+a)**3/x,x)
```

output
```
Integral(cosh(a + b*x**2)**3/x, x)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\cosh^3(a+bx^2)}{x}\,dx = \frac{1}{16}\,\mathrm{Ei}\left(3\,bx^2\right)e^{(3\,a)} + \frac{3}{16}\,\mathrm{Ei}\left(-bx^2\right)e^{(-a)} + \frac{1}{16}\,\mathrm{Ei}\left(-3\,bx^2\right)e^{(-3\,a)} + \frac{3}{16}\,\mathrm{Ei}\left(bx^2\right)e^{a}$$

input
```
integrate(cosh(b*x^2+a)^3/x,x, algorithm="maxima")
```

output
```
1/16*Ei(3*b*x^2)*e^(3*a) + 3/16*Ei(-b*x^2)*e^(-a) + 1/16*Ei(-3*b*x^2)*e^(-
3*a) + 3/16*Ei(b*x^2)*e^a
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\cosh^3(a+bx^2)}{x}\,dx = \frac{1}{16}\,\mathrm{Ei}\left(3\,bx^2\right)e^{(3\,a)} + \frac{3}{16}\,\mathrm{Ei}\left(-bx^2\right)e^{(-a)} + \frac{1}{16}\,\mathrm{Ei}\left(-3\,bx^2\right)e^{(-3\,a)} + \frac{3}{16}\,\mathrm{Ei}\left(bx^2\right)e^{a}$$

input
```
integrate(cosh(b*x^2+a)^3/x,x, algorithm="giac")
```

output
```
1/16*Ei(3*b*x^2)*e^(3*a) + 3/16*Ei(-b*x^2)*e^(-a) + 1/16*Ei(-3*b*x^2)*e^(-
3*a) + 3/16*Ei(b*x^2)*e^a
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx^2)}{x}\,dx=\int \frac{{\cosh\left(b\,x^2+a\right)}^3}{x}\,dx$$

input
```
int(cosh(a + b*x^2)^3/x,x)
```

output
```
int(cosh(a + b*x^2)^3/x, x)
```

3.20 $\int \frac{\cosh^3(a+bx^2)}{x^2}\,dx$

3.20.1 Optimal result

Integrand size = 14, antiderivative size = 136

$$\begin{aligned}\int \frac{\cosh^3(a+bx^2)}{x^2}\,dx = &-\frac{\cosh^3(a+bx^2)}{x}-\frac{3}{8}\sqrt{b}e^{-a}\sqrt{\pi}\,\text{erf}\left(\sqrt{b}x\right)\\ &-\frac{1}{8}\sqrt{b}e^{-3a}\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\sqrt{b}x\right)\\ &+\frac{3}{8}\sqrt{b}e^{a}\sqrt{\pi}\,\text{erfi}\left(\sqrt{b}x\right)+\frac{1}{8}\sqrt{b}e^{3a}\sqrt{3\pi}\,\text{erfi}\left(\sqrt{3}\sqrt{b}x\right)\end{aligned}$$

output
```
-cosh(b*x^2+a)^3/x-3/8*erf(x*b^(1/2))*b^(1/2)*Pi^(1/2)/exp(a)+3/8*exp(a)*e
rfi(x*b^(1/2))*b^(1/2)*Pi^(1/2)-1/8*erf(x*3^(1/2)*b^(1/2))*b^(1/2)*3^(1/2)
*Pi^(1/2)/exp(3*a)+1/8*exp(3*a)*erfi(x*3^(1/2)*b^(1/2))*b^(1/2)*3^(1/2)*Pi
^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.50

$$\int \frac{\cosh^3(a+bx^2)}{x^2}\,dx$$

$$= \frac{-6\cosh(a+bx^2)-2\cosh(3(a+bx^2))+3\sqrt{b}\sqrt{\pi}x\cosh(a)\text{erfi}\left(\sqrt{b}x\right)+\sqrt{b}\sqrt{3\pi}x\cosh(3a)\text{erfi}\left(\sqrt{3}\sqrt{b}x\right)}{}$$

input
```
Integrate[Cosh[a + b*x^2]^3/x^2,x]
```

output
```
(-6*Cosh[a + b*x^2] - 2*Cosh[3*(a + b*x^2)] + 3*Sqrt[b]*Sqrt[Pi]*x*Cosh[a]
*Erfi[Sqrt[b]*x] + Sqrt[b]*Sqrt[3*Pi]*x*Cosh[3*a]*Erfi[Sqrt[3]*Sqrt[b]*x]
+ 3*Sqrt[b]*Sqrt[Pi]*x*Erfi[Sqrt[b]*x]*Sinh[a] + 3*Sqrt[b]*Sqrt[Pi]*x*Erf[
Sqrt[b]*x]*(-Cosh[a] + Sinh[a]) + Sqrt[b]*Sqrt[3*Pi]*x*Erfi[Sqrt[3]*Sqrt[b
]*x]*Sinh[3*a] + Sqrt[b]*Sqrt[3*Pi]*x*Erf[Sqrt[3]*Sqrt[b]*x]*(-Cosh[3*a] +
 Sinh[3*a]))/(8*x)
```

3.20.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5854, 6152, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(a+bx^2)}{x^2}\,dx$$

$$\Big\downarrow \text{5854}$$

$$6b\int \cosh^2(bx^2+a)\sinh(bx^2+a)\,dx-\frac{\cosh^3(a+bx^2)}{x}$$

$$\Big\downarrow \text{6152}$$

$$6b\int \left(\frac{1}{4}\sinh(bx^2+a)+\frac{1}{4}\sinh(3bx^2+3a)\right)dx-\frac{\cosh^3(a+bx^2)}{x}$$

$$\Big\downarrow \text{2009}$$

$$6b\left(-\frac{\sqrt{\pi}e^{-a}\text{erf}\left(\sqrt{b}x\right)}{16\sqrt{b}}-\frac{\sqrt{\frac{\pi}{3}}e^{-3a}\text{erf}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}}+\frac{\sqrt{\pi}e^{a}\text{erfi}\left(\sqrt{b}x\right)}{16\sqrt{b}}+\frac{\sqrt{\frac{\pi}{3}}e^{3a}\text{erfi}\left(\sqrt{3}\sqrt{b}x\right)}{16\sqrt{b}}\right)-\frac{\cosh^3(a+bx^2)}{x}$$

input
```
Int[Cosh[a + b*x^2]^3/x^2,x]
```

output
```
-(Cosh[a + b*x^2]^3/x) + 6*b*(-1/16*(Sqrt[Pi]*Erf[Sqrt[b]*x])/(Sqrt[b]*E^a
) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[b]*x])/(16*Sqrt[b]*E^(3*a)) + (E^a*Sqrt[P
i]*Erfi[Sqrt[b]*x])/(16*Sqrt[b]) + (E^(3*a)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[b
]*x])/(16*Sqrt[b]))
```

3.20.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5854
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_)]^(p_)*(x_)^(m_.), x_Symbol] :> Simp[-Cosh[
a + b*x^n]^p/((n - 1)*x^(n - 1)), x] + Simp[b*n*(p/(n - 1))   Int[Cosh[a +
b*x^n]^(p - 1)*Sinh[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IntegersQ[n,
p] && EqQ[m + n, 0] && GtQ[p, 1] && NeQ[n, 1]
```

rule 6152
```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

3.20.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

method	result
risch	$-\frac{e^{-3a} e^{-3b x^2}}{8x} - \frac{e^{-3a}\sqrt{b}\sqrt{\pi}\sqrt{3}\,\mathrm{erf}\left(x\sqrt{3}\sqrt{b}\right)}{8} - \frac{3\,e^{-a} e^{-b x^2}}{8x} - \frac{3\,\mathrm{erf}\left(x\sqrt{b}\right)\sqrt{b}\sqrt{\pi}\,e^{-a}}{8} - \frac{e^{3a} e^{3b x^2}}{8x} + \frac{3\,e^{3a} b\sqrt{\pi}\,\mathrm{erf}(\sqrt{-3b}}{8\sqrt{-3b}}$

input
```
int(cosh(b*x^2+a)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8/exp(a)^3/x*exp(-3*b*x^2)-1/8/exp(a)^3*b^(1/2)*Pi^(1/2)*3^(1/2)*erf(x*
3^(1/2)*b^(1/2))-3/8/exp(a)/x*exp(-b*x^2)-3/8*erf(x*b^(1/2))*b^(1/2)*Pi^(1
/2)/exp(a)-1/8*exp(a)^3/x*exp(3*b*x^2)+3/8*exp(a)^3*b*Pi^(1/2)/(-3*b)^(1/2
)*erf((-3*b)^(1/2)*x)-3/8*exp(a)*exp(b*x^2)/x+3/8*exp(a)*b*Pi^(1/2)/(-b)^(
1/2)*erf((-b)^(1/2)*x)
```

3.20.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 891 vs. $2(98) = 196$.

Time = 0.26 (sec) , antiderivative size = 891, normalized size of antiderivative = 6.55

$$\int \frac{\cosh^3(a+bx^2)}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x^2+a)^3/x^2,x, algorithm="fricas")
```

output
```
-1/8*(cosh(b*x^2 + a)^6 + 6*cosh(b*x^2 + a)*sinh(b*x^2 + a)^5 + sinh(b*x^2
 + a)^6 + 3*(5*cosh(b*x^2 + a)^2 + 1)*sinh(b*x^2 + a)^4 + 3*cosh(b*x^2 + a
)^4 + 4*(5*cosh(b*x^2 + a)^3 + 3*cosh(b*x^2 + a))*sinh(b*x^2 + a)^3 + sqrt
(3)*sqrt(pi)*(x*cosh(b*x^2 + a)^3*cosh(3*a) + x*cosh(b*x^2 + a)^3*sinh(3*a
) + (x*cosh(3*a) + x*sinh(3*a))*sinh(b*x^2 + a)^3 + 3*(x*cosh(b*x^2 + a)*c
osh(3*a) + x*cosh(b*x^2 + a)*sinh(3*a))*sinh(b*x^2 + a)^2 + 3*(x*cosh(b*x^
2 + a)^2*cosh(3*a) + x*cosh(b*x^2 + a)^2*sinh(3*a))*sinh(b*x^2 + a))*sqrt(
-b)*erf(sqrt(3)*sqrt(-b)*x) + sqrt(3)*sqrt(pi)*(x*cosh(b*x^2 + a)^3*cosh(3
*a) - x*cosh(b*x^2 + a)^3*sinh(3*a) + (x*cosh(3*a) - x*sinh(3*a))*sinh(b*x
^2 + a)^3 + 3*(x*cosh(b*x^2 + a)*cosh(3*a) - x*cosh(b*x^2 + a)*sinh(3*a))*
sinh(b*x^2 + a)^2 + 3*(x*cosh(b*x^2 + a)^2*cosh(3*a) - x*cosh(b*x^2 + a)^2
*sinh(3*a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(3)*sqrt(b)*x) + 3*sqrt(pi)*(
x*cosh(b*x^2 + a)^3*cosh(a) + x*cosh(b*x^2 + a)^3*sinh(a) + (x*cosh(a) + x
*sinh(a))*sinh(b*x^2 + a)^3 + 3*(x*cosh(b*x^2 + a)*cosh(a) + x*cosh(b*x^2
+ a)*sinh(a))*sinh(b*x^2 + a)^2 + 3*(x*cosh(b*x^2 + a)^2*cosh(a) + x*cosh(
b*x^2 + a)^2*sinh(a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(-b)*x) + 3*sqrt(p
i)*(x*cosh(b*x^2 + a)^3*cosh(a) - x*cosh(b*x^2 + a)^3*sinh(a) + (x*cosh(a)
 - x*sinh(a))*sinh(b*x^2 + a)^3 + 3*(x*cosh(b*x^2 + a)*cosh(a) - x*cosh(b*
x^2 + a)*sinh(a))*sinh(b*x^2 + a)^2 + 3*(x*cosh(b*x^2 + a)^2*cosh(a) - x*c
osh(b*x^2 + a)^2*sinh(a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(b)*x) + 3*(...
```

3.20.6 Sympy [F]

$$\int \frac{\cosh^3(a+bx^2)}{x^2}\,dx = \int \frac{\cosh^3(a+bx^2)}{x^2}\,dx$$

input
```
integrate(cosh(b*x**2+a)**3/x**2,x)
```

output
```
Integral(cosh(a + b*x**2)**3/x**2, x)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.75

$$\int \frac{\cosh^3(a+bx^2)}{x^2}\,dx = -\frac{\sqrt{3}\sqrt{bx^2}e^{(-3a)}\Gamma\left(-\frac{1}{2},3bx^2\right)}{16x} - \frac{\sqrt{3}\sqrt{-bx^2}e^{(3a)}\Gamma\left(-\frac{1}{2},-3bx^2\right)}{16x} - \frac{3\sqrt{bx^2}e^{(-a)}\Gamma\left(-\frac{1}{2},bx^2\right)}{16x} - \frac{3\sqrt{-bx^2}e^{a}\Gamma\left(-\frac{1}{2},-bx^2\right)}{16x}$$

input
```
integrate(cosh(b*x^2+a)^3/x^2,x, algorithm="maxima")
```

output
```
-1/16*sqrt(3)*sqrt(b*x^2)*e^(-3*a)*gamma(-1/2, 3*b*x^2)/x - 1/16*sqrt(3)*s
qrt(-b*x^2)*e^(3*a)*gamma(-1/2, -3*b*x^2)/x - 3/16*sqrt(b*x^2)*e^(-a)*gamm
a(-1/2, b*x^2)/x - 3/16*sqrt(-b*x^2)*e^a*gamma(-1/2, -b*x^2)/x
```

3.20.8 Giac [F]

$$\int \frac{\cosh^3(a+bx^2)}{x^2}\,dx = \int \frac{\cosh(bx^2+a)^3}{x^2}\,dx$$

input
```
integrate(cosh(b*x^2+a)^3/x^2,x, algorithm="giac")
```

output
```
integrate(cosh(b*x^2 + a)^3/x^2, x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx^2)}{x^2}\,dx = \int \frac{\cosh(b\,x^2+a)^3}{x^2}\,dx$$

input
```
int(cosh(a + b*x^2)^3/x^2,x)
```

output
```
int(cosh(a + b*x^2)^3/x^2, x)
```

3.21 $\int \frac{\cosh^3(a+bx^2)}{x^3} \, dx$

3.21.1 Optimal result

Integrand size = 14, antiderivative size = 91

$$\int \frac{\cosh^3(a+bx^2)}{x^3} \, dx = -\frac{3\cosh(a+bx^2)}{8x^2} - \frac{\cosh(3(a+bx^2))}{8x^2} + \frac{3}{8} b \text{Chi}(bx^2) \sinh(a) + \frac{3}{8} b \text{Chi}(3bx^2) \sinh(3a) + \frac{3}{8} b \cosh(a) \text{Shi}(bx^2) + \frac{3}{8} b \cosh(3a) \text{Shi}(3bx^2)$$

output
```
-3/8*cosh(b*x^2+a)/x^2-1/8*cosh(3*b*x^2+3*a)/x^2+3/8*b*cosh(a)*Shi(b*x^2)+
3/8*b*cosh(3*a)*Shi(3*b*x^2)+3/8*b*Chi(b*x^2)*sinh(a)+3/8*b*Chi(3*b*x^2)*s
inh(3*a)
```

3.21.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

$$\int \frac{\cosh^3(a+bx^2)}{x^3} \, dx = \frac{-3\cosh(a+bx^2) - \cosh(3(a+bx^2)) + 3bx^2 \text{Chi}(bx^2) \sinh(a) + 3bx^2 \text{Chi}(3bx^2) \sinh(3a) + 3bx^2 \cosh(a) \text{Sl}}{8x^2}$$

input
```
Integrate[Cosh[a + b*x^2]^3/x^3,x]
```

output
```
(-3*Cosh[a + b*x^2] - Cosh[3*(a + b*x^2)] + 3*b*x^2*CoshIntegral[b*x^2]*Si
nh[a] + 3*b*x^2*CoshIntegral[3*b*x^2]*Sinh[3*a] + 3*b*x^2*Cosh[a]*SinhInte
gral[b*x^2] + 3*b*x^2*Cosh[3*a]*SinhIntegral[3*b*x^2])/(8*x^2)
```

3.21.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5864, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3\left(a+bx^2\right)}{x^3}dx$$

$$\Big\downarrow \text{5864}$$

$$\int \left(\frac{3\cosh\left(a+bx^2\right)}{4x^3}+\frac{\cosh\left(3a+3bx^2\right)}{4x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{8}b\sinh(a)\text{Chi}\left(bx^2\right)+\frac{3}{8}b\sinh(3a)\text{Chi}\left(3bx^2\right)+\frac{3}{8}b\cosh(a)\text{Shi}\left(bx^2\right)+\frac{3}{8}b\cosh(3a)\text{Shi}\left(3bx^2\right)-\frac{3\cosh\left(a+bx^2\right)}{8x^2}-\frac{\cosh\left(3\left(a+bx^2\right)\right)}{8x^2}$$

input
```
Int[Cosh[a + b*x^2]^3/x^3,x]
```

output
```
(-3*Cosh[a + b*x^2])/(8*x^2) - Cosh[3*(a + b*x^2)]/(8*x^2) + (3*b*CoshInte
gral[b*x^2]*Sinh[a])/8 + (3*b*CoshIntegral[3*b*x^2]*Sinh[3*a])/8 + (3*b*Co
sh[a]*SinhIntegral[b*x^2])/8 + (3*b*Cosh[3*a]*SinhIntegral[3*b*x^2])/8
```

3.21.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5864
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

3.21.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{-3\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1\left(3b\,x^2\right)b\,x^2-3\,\mathrm{e}^{-a}\,\mathrm{Ei}_1\left(b\,x^2\right)b\,x^2+3\,\mathrm{e}^{3a}\,\mathrm{Ei}_1\left(-3b\,x^2\right)b\,x^2+3\,\mathrm{Ei}_1\left(-b\,x^2\right)\mathrm{e}^{a}b\,x^2+\mathrm{e}^{-3b\,x^2-3a}+3\,\mathrm{e}^{-b\,x^2-a}+\mathrm{e}^{3b\,x^2+3a}+3}{16x^2}$

input
```
int(cosh(b*x^2+a)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-3*exp(-3*a)*Ei(1,3*b*x^2)*b*x^2-3*exp(-a)*Ei(1,b*x^2)*b*x^2+3*exp(
3*a)*Ei(1,-3*b*x^2)*b*x^2+3*Ei(1,-b*x^2)*exp(a)*b*x^2+exp(-3*b*x^2-3*a)+3*
exp(-b*x^2-a)+exp(3*b*x^2+3*a)+3*exp(b*x^2+a))/x^2
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(80) = 160.

Time = 0.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.85

$$\int \frac{\cosh^3(a+bx^2)}{x^3}\,dx =$$
$$-\frac{2\cosh\left(bx^2+a\right)^3+6\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^2-3\left(bx^2\mathrm{Ei}(3\,bx^2)-bx^2\mathrm{Ei}(-3\,bx^2)\right)\cosh\left(3\,a\right)-3\,(}{}$$

input
```
integrate(cosh(b*x^2+a)^3/x^3,x, algorithm="fricas")
```

output
```
-1/16*(2*cosh(b*x^2 + a)^3 + 6*cosh(b*x^2 + a)*sinh(b*x^2 + a)^2 - 3*(b*x^
2*Ei(3*b*x^2) - b*x^2*Ei(-3*b*x^2))*cosh(3*a) - 3*(b*x^2*Ei(b*x^2) - b*x^2
*Ei(-b*x^2))*cosh(a) - 3*(b*x^2*Ei(3*b*x^2) + b*x^2*Ei(-3*b*x^2))*sinh(3*a
) - 3*(b*x^2*Ei(b*x^2) + b*x^2*Ei(-b*x^2))*sinh(a) + 6*cosh(b*x^2 + a))/x^
2
```

3.21.6 Sympy [F]

$$\int \frac{\cosh^3(a+bx^2)}{x^3}\,dx = \int \frac{\cosh^3(a+bx^2)}{x^3}\,dx$$

input
```
integrate(cosh(b*x**2+a)**3/x**3,x)
```

output
```
Integral(cosh(a + b*x**2)**3/x**3, x)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

$$\int \frac{\cosh^3(a+bx^2)}{x^3}\,dx = -\frac{3}{16}\,be^{(-3\,a)}\Gamma\left(-1,3\,bx^2\right) - \frac{3}{16}\,be^{(-a)}\Gamma\left(-1,bx^2\right) + \frac{3}{16}\,be^{a}\Gamma\left(-1,-bx^2\right) + \frac{3}{16}\,be^{(3\,a)}\Gamma\left(-1,-3\,bx^2\right)$$

input
```
integrate(cosh(b*x^2+a)^3/x^3,x, algorithm="maxima")
```

output
```
-3/16*b*e^(-3*a)*gamma(-1, 3*b*x^2) - 3/16*b*e^(-a)*gamma(-1, b*x^2) + 3/1
6*b*e^a*gamma(-1, -b*x^2) + 3/16*b*e^(3*a)*gamma(-1, -3*b*x^2)
```

3.21.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(80) = 160$.

Time = 0.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.46

$$\int \frac{\cosh^3(a+bx^2)}{x^3}\,dx$$
$$= \frac{3\,(bx^2+a)b^2\text{Ei}(3\,bx^2)\,e^{(3\,a)} - 3\,ab^2\text{Ei}(3\,bx^2)\,e^{(3\,a)} - 3\,(bx^2+a)b^2\text{Ei}(-bx^2)\,e^{(-a)} + 3\,ab^2\text{Ei}(-bx^2)\,e^{(-a)} - 3}{}$$

input
```
integrate(cosh(b*x^2+a)^3/x^3,x, algorithm="giac")
```

output
```
1/16*(3*(b*x^2 + a)*b^2*Ei(3*b*x^2)*e^(3*a) - 3*a*b^2*Ei(3*b*x^2)*e^(3*a)
- 3*(b*x^2 + a)*b^2*Ei(-b*x^2)*e^(-a) + 3*a*b^2*Ei(-b*x^2)*e^(-a) - 3*(b*x
^2 + a)*b^2*Ei(-3*b*x^2)*e^(-3*a) + 3*a*b^2*Ei(-3*b*x^2)*e^(-3*a) + 3*(b*x
^2 + a)*b^2*Ei(b*x^2)*e^a - 3*a*b^2*Ei(b*x^2)*e^a - b^2*e^(3*b*x^2 + 3*a)
- 3*b^2*e^(b*x^2 + a) - 3*b^2*e^(-b*x^2 - a) - b^2*e^(-3*b*x^2 - 3*a))/(b^
2*x^2)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx^2)}{x^3}\,dx = \int \frac{\cosh(b\,x^2+a)^3}{x^3}\,dx$$

input
```
int(cosh(a + b*x^2)^3/x^3,x)
```

output
```
int(cosh(a + b*x^2)^3/x^3, x)
```

3.22 $\int x \cosh^7(a+bx^2)\,dx$

3.22.1 Optimal result

Integrand size = 12, antiderivative size = 67

$$\int x\cosh^7\left(a+bx^2\right)\,dx=\frac{\sinh\left(a+bx^2\right)}{2b}+\frac{\sinh^3\left(a+bx^2\right)}{2b}+\frac{3\sinh^5\left(a+bx^2\right)}{10b}+\frac{\sinh^7\left(a+bx^2\right)}{14b}$$

output
```
1/2*sinh(b*x^2+a)/b+1/2*sinh(b*x^2+a)^3/b+3/10*sinh(b*x^2+a)^5/b+1/14*sinh
(b*x^2+a)^7/b
```

3.22.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int x\cosh^7\left(a+bx^2\right)\,dx=\frac{\sinh\left(a+bx^2\right)}{2b}+\frac{\sinh^3\left(a+bx^2\right)}{2b}+\frac{3\sinh^5\left(a+bx^2\right)}{10b}+\frac{\sinh^7\left(a+bx^2\right)}{14b}$$

input
```
Integrate[x*Cosh[a + b*x^2]^7,x]
```

output
```
Sinh[a + b*x^2]/(2*b) + Sinh[a + b*x^2]^3/(2*b) + (3*Sinh[a + b*x^2]^5)/(1
0*b) + Sinh[a + b*x^2]^7/(14*b)
```

3.22.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5844, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh^7\left(a+bx^2\right)\,dx$$

$$\Big\downarrow \text{5844}$$

$$\frac{1}{2}\int \cosh^7\left(bx^2+a\right)dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin\left(ibx^2+ia+\frac{\pi}{2}\right)^7dx^2$$

$$\Big\downarrow \text{3113}$$

$$\frac{i\int\left(-x^{12}+3x^8-3x^4+1\right)d\left(-i\sinh\left(bx^2+a\right)\right)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-i\sinh\left(a+bx^2\right)-\frac{x^{14}}{7}+\frac{3x^{10}}{5}-x^6\right)}{2b}$$

input
```
Int[x*Cosh[a + b*x^2]^7,x]
```

output
```
((I/2)*(-x^6 + (3*x^10)/5 - x^14/7 - I*Sinh[a + b*x^2]))/b
```

3.22.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

3.22.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\left(\frac{16}{35}+\frac{\cosh\left(b\,x^{2}+a\right)^{6}}{7}+\frac{6\cosh\left(b\,x^{2}+a\right)^{4}}{35}+\frac{8\cosh\left(b\,x^{2}+a\right)^{2}}{35}\right)\sinh\left(b\,x^{2}+a\right)}{2b}$
default	$\frac{\left(\frac{16}{35}+\frac{\cosh\left(b\,x^{2}+a\right)^{6}}{7}+\frac{6\cosh\left(b\,x^{2}+a\right)^{4}}{35}+\frac{8\cosh\left(b\,x^{2}+a\right)^{2}}{35}\right)\sinh\left(b\,x^{2}+a\right)}{2b}$
parallelrisch	$\frac{5\sinh\left(7b\,x^{2}+7a\right)+49\sinh\left(5b\,x^{2}+5a\right)+245\sinh\left(3b\,x^{2}+3a\right)+1225\sinh\left(b\,x^{2}+a\right)}{4480b}$
risch	$\frac{{\mathrm e}^{7b\,x^{2}+7a}}{1792b}+\frac{7\,{\mathrm e}^{5b\,x^{2}+5a}}{1280b}+\frac{7\,{\mathrm e}^{3b\,x^{2}+3a}}{256b}+\frac{35\,{\mathrm e}^{b\,x^{2}+a}}{256b}-\frac{35\,{\mathrm e}^{-b\,x^{2}-a}}{256b}-\frac{7\,{\mathrm e}^{-3b\,x^{2}-3a}}{256b}-\frac{7\,{\mathrm e}^{-5b\,x^{2}-5a}}{1280b}-\frac{{\mathrm e}^{-7b\,x^{2}}}{1792}$

input
```
int(x*cosh(b*x^2+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(16/35+1/7*cosh(b*x^2+a)^6+6/35*cosh(b*x^2+a)^4+8/35*cosh(b*x^2+a)^2
)*sinh(b*x^2+a)
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(59) = 118.

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.91

$$\int x\cosh^7\left(a+bx^2\right)\,dx$$
$$=\frac{5\,\sinh\left(bx^2+a\right)^7+7\left(15\,\cosh\left(bx^2+a\right)^2+7\right)\sinh\left(bx^2+a\right)^5+35\left(5\,\cosh\left(bx^2+a\right)^4+14\,\cosh\left(bx^2+a\right.\right.}{44}$$

input
```
integrate(x*cosh(b*x^2+a)^7,x, algorithm="fricas")
```

output
```
1/4480*(5*sinh(b*x^2 + a)^7 + 7*(15*cosh(b*x^2 + a)^2 + 7)*sinh(b*x^2 + a)
^5 + 35*(5*cosh(b*x^2 + a)^4 + 14*cosh(b*x^2 + a)^2 + 7)*sinh(b*x^2 + a)^3
 + 35*(cosh(b*x^2 + a)^6 + 7*cosh(b*x^2 + a)^4 + 21*cosh(b*x^2 + a)^2 + 35
)*sinh(b*x^2 + a))/b
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int x\cosh^7\left(a+bx^2\right)\,dx$$
$$=\begin{cases}-\frac{8\sinh^7\left(a+bx^2\right)}{35b}+\frac{4\sinh^5\left(a+bx^2\right)\cosh^2\left(a+bx^2\right)}{5b}-\frac{\sinh^3\left(a+bx^2\right)\cosh^4\left(a+bx^2\right)}{b}+\frac{\sinh\left(a+bx^2\right)\cosh^6\left(a+bx^2\right)}{2b} & \text{for } b\neq 0\\ \frac{x^2\cosh^7\left(a\right)}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*cosh(b*x**2+a)**7,x)
```

output
```
Piecewise((-8*sinh(a + b*x**2)**7/(35*b) + 4*sinh(a + b*x**2)**5*cosh(a +
b*x**2)**2/(5*b) - sinh(a + b*x**2)**3*cosh(a + b*x**2)**4/b + sinh(a + b*
x**2)*cosh(a + b*x**2)**6/(2*b), Ne(b, 0)), (x**2*cosh(a)**7/2, True))
```

3.22.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(59) = 118$.

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.88

$$\int x\cosh^7\left(a+bx^2\right)\,dx = \frac{e^{\left(7\,bx^2+7\,a\right)}}{1792\,b}+\frac{7\,e^{\left(5\,bx^2+5\,a\right)}}{1280\,b}+\frac{7\,e^{\left(3\,bx^2+3\,a\right)}}{256\,b}+\frac{35\,e^{\left(bx^2+a\right)}}{256\,b}-\frac{35\,e^{\left(-bx^2-a\right)}}{256\,b}-\frac{7\,e^{\left(-3\,bx^2-3\,a\right)}}{256\,b}-\frac{7\,e^{\left(-5\,bx^2-5\,a\right)}}{1280\,b}-\frac{e^{\left(-7\,bx^2-7\,a\right)}}{1792\,b}$$

input
```
integrate(x*cosh(b*x^2+a)^7,x, algorithm="maxima")
```

output
```
1/1792*e^(7*b*x^2 + 7*a)/b + 7/1280*e^(5*b*x^2 + 5*a)/b + 7/256*e^(3*b*x^2
 + 3*a)/b + 35/256*e^(b*x^2 + a)/b - 35/256*e^(-b*x^2 - a)/b - 7/256*e^(-3
*b*x^2 - 3*a)/b - 7/1280*e^(-5*b*x^2 - 5*a)/b - 1/1792*e^(-7*b*x^2 - 7*a)/
b
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.61

$$\int x\cosh^7\left(a+bx^2\right)\,dx = -\frac{\left(1225\,e^{\left(6\,bx^2+6\,a\right)}+245\,e^{\left(4\,bx^2+4\,a\right)}+49\,e^{\left(2\,bx^2+2\,a\right)}+5\right)e^{\left(-7\,bx^2-7\,a\right)}-5\,e^{\left(7\,bx^2+7\,a\right)}-49\,e^{\left(5\,bx^2+5\,a\right)}-245\,e}{8960\,b}$$

input
```
integrate(x*cosh(b*x^2+a)^7,x, algorithm="giac")
```

output
```
-1/8960*((1225*e^(6*b*x^2 + 6*a) + 245*e^(4*b*x^2 + 4*a) + 49*e^(2*b*x^2 +
 2*a) + 5)*e^(-7*b*x^2 - 7*a) - 5*e^(7*b*x^2 + 7*a) - 49*e^(5*b*x^2 + 5*a)
 - 245*e^(3*b*x^2 + 3*a) - 1225*e^(b*x^2 + a))/b
```

3.22.9 Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int x\cosh^7\left(a+bx^2\right)\,dx$$
$$=\frac{5\sinh(b\,x^2+a)^7+21\sinh(b\,x^2+a)^5+35\sinh(b\,x^2+a)^3+35\sinh(b\,x^2+a)}{70\,b}$$

input
```
int(x*cosh(a + b*x^2)^7,x)
```

output
```
(35*sinh(a + b*x^2) + 35*sinh(a + b*x^2)^3 + 21*sinh(a + b*x^2)^5 + 5*sinh
(a + b*x^2)^7)/(70*b)
```

3.23 $\int x^2 \cosh(x^3)\, dx$

3.23.1 Optimal result

Integrand size = 8, antiderivative size = 8

$$\int x^2 \cosh(x^3)\, dx = \frac{\sinh(x^3)}{3}$$

output
```
1/3*sinh(x^3)
```

3.23.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x^2 \cosh(x^3)\, dx = \frac{\sinh(x^3)}{3}$$

input
```
Integrate[x^2*Cosh[x^3],x]
```

output
```
Sinh[x^3]/3
```

3.23.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5844, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cosh \left(x^3\right) \, dx$$

$$\Big\downarrow \text{ 5844}$$

$$\frac{1}{3} \int \cosh \left(x^3\right) dx^3$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3} \int \sin \left(i x^3 + \frac{\pi}{2}\right) dx^3$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{\sinh \left(x^3\right)}{3}$$

input
```
Int[x^2*Cosh[x^3],x]
```

output
```
Sinh[x^3]/3
```

3.23.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

3.23.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\sinh(x^3)}{3}$	7
default	$\frac{\sinh(x^3)}{3}$	7
meijerg	$\frac{\sinh(x^3)}{3}$	7
parallelrisch	$\frac{\sinh(x^3)}{3}$	7
risch	$\frac{e^{x^3}}{6} - \frac{e^{-x^3}}{6}$	16

input
```
int(x^2*cosh(x^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*sinh(x^3)
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x^2 \cosh\left(x^3\right)\, dx = \frac{1}{3}\sinh\left(x^3\right)$$

input
```
integrate(x^2*cosh(x^3),x, algorithm="fricas")
```

output
```
1/3*sinh(x^3)
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int x^2 \cosh\left(x^3\right)\,dx = \frac{\sinh\left(x^3\right)}{3}$$

input
```
integrate(x**2*cosh(x**3),x)
```

output
```
sinh(x**3)/3
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x^2 \cosh\left(x^3\right)\,dx = \frac{1}{3}\sinh\left(x^3\right)$$

input
```
integrate(x^2*cosh(x^3),x, algorithm="maxima")
```

output
```
1/3*sinh(x^3)
```

3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(6) = 12.

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int x^2 \cosh\left(x^3\right)\,dx = -\frac{1}{6}\,e^{\left(-x^3\right)} + \frac{1}{6}\,e^{\left(x^3\right)}$$

input
```
integrate(x^2*cosh(x^3),x, algorithm="giac")
```

output
```
-1/6*e^(-x^3) + 1/6*e^(x^3)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x^2 \cosh\left(x^3\right) \, dx = \frac{\sinh(x^3)}{3}$$

input
```
int(x^2*cosh(x^3),x)
```

output
```
sinh(x^3)/3
```

3.24 $\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6}\,dx$

3.24.1 Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6}\,dx = -\frac{1}{5}\sinh\left(\frac{1}{x^5}\right)$$

output
```
-1/5*sinh(1/x^5)
```

3.24.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6}\,dx = -\frac{1}{5}\sinh\left(\frac{1}{x^5}\right)$$

input
```
Integrate[Cosh[x^(-5)]/x^6,x]
```

output
```
-1/5*Sinh[x^(-5)]
```

3.24.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5844, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6}dx$$

$$\Big\downarrow \text{5844}$$

$$-\frac{1}{5}\int \cosh\left(\frac{1}{x^5}\right)d\frac{1}{x^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5}\int \sin\left(\frac{\pi}{2}+\frac{i}{x^5}\right)d\frac{1}{x^5}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{1}{5}\sinh\left(\frac{1}{x^5}\right)$$

input
```
Int[Cosh[x^(-5)]/x^6,x]
```

output
```
-1/5*Sinh[x^(-5)]
```

3.24.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

3.24.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\sinh\left(\frac{1}{x^5}\right)}{5}$	7
default	$-\frac{\sinh\left(\frac{1}{x^5}\right)}{5}$	7
meijerg	$-\frac{\sinh\left(\frac{1}{x^5}\right)}{5}$	7
parallelrisch	$-\frac{\sinh\left(\frac{1}{x^5}\right)}{5}$	7
risch	$-\frac{e^{\frac{1}{x^5}}}{10}+\frac{e^{-\frac{1}{x^5}}}{10}$	16

input
```
int(cosh(1/x^5)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*sinh(1/x^5)
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6}\,dx = -\frac{1}{5}\sinh\left(\frac{1}{x^5}\right)$$

input
```
integrate(cosh(1/x^5)/x^6,x, algorithm="fricas")
```

output
```
-1/5*sinh(x^(-5))
```

3.24.6 Sympy [A] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6}\,dx = -\frac{\sinh\left(\frac{1}{x^5}\right)}{5}$$

input
```
integrate(cosh(1/x**5)/x**6,x)
```

output
```
-sinh(x**(-5))/5
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6}\,dx = -\frac{1}{5}\sinh\left(\frac{1}{x^5}\right)$$

input
```
integrate(cosh(1/x^5)/x^6,x, algorithm="maxima")
```

output
```
-1/5*sinh(x^(-5))
```

3.24.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(6) = 12.

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6}\,dx = \frac{1}{10}e^{\left(-\frac{1}{x^5}\right)} - \frac{1}{10}e^{\left(\frac{1}{x^5}\right)}$$

input
```
integrate(cosh(1/x^5)/x^6,x, algorithm="giac")
```

output
```
1/10*e^(-1/x^5) - 1/10*e^(x^(-5))
```

3.24.9 Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6}\,dx = \frac{e^{-\frac{1}{x^5}}}{10} - \frac{e^{\frac{1}{x^5}}}{10}$$

input
```
int(cosh(1/x^5)/x^6,x)
```

output
```
exp(-1/x^5)/10 - exp(1/x^5)/10
```

3.25 $\int \cosh \left(a+\frac{b}{x}\right) dx$

3.25.1 Optimal result

Integrand size = 8, antiderivative size = 33

$$\int \cosh \left(a+\frac{b}{x}\right) dx = x\cosh \left(a+\frac{b}{x}\right) - b\text{Chi}\left(\frac{b}{x}\right)\sinh(a) - b\cosh(a)\text{Shi}\left(\frac{b}{x}\right)$$

output
```
x*cosh(a+b/x)-b*cosh(a)*Shi(b/x)-b*Chi(b/x)*sinh(a)
```

3.25.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \cosh \left(a+\frac{b}{x}\right) dx = x\cosh \left(a+\frac{b}{x}\right) - b\text{Chi}\left(\frac{b}{x}\right)\sinh(a) - b\cosh(a)\text{Shi}\left(\frac{b}{x}\right)$$

input
```
Integrate[Cosh[a + b/x],x]
```

output
```
x*Cosh[a + b/x] - b*CoshIntegral[b/x]*Sinh[a] - b*Cosh[a]*SinhIntegral[b/x
]
```

3.25.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {5826, 3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh\left(a+\frac{b}{x}\right)dx$$

$$\Big\downarrow \text{5826}$$

$$-\int x^2\cosh\left(a+\frac{b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int x^2\sin\left(ia+\frac{ib}{x}+\frac{\pi}{2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3778}$$

$$x\cosh\left(a+\frac{b}{x}\right)-ib\int -ix\sinh\left(a+\frac{b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{26}$$

$$x\cosh\left(a+\frac{b}{x}\right)-b\int x\sinh\left(a+\frac{b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$x\cosh\left(a+\frac{b}{x}\right)-b\int -ix\sin\left(ia+\frac{ib}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{26}$$

$$x\cosh\left(a+\frac{b}{x}\right)+ib\int x\sin\left(ia+\frac{ib}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3784}$$

$$x\cosh\left(a+\frac{b}{x}\right)+ib\left(i\sinh(a)\int x\cosh\left(\frac{b}{x}\right)d\frac{1}{x}+\cosh(a)\int ix\sinh\left(\frac{b}{x}\right)d\frac{1}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$x\cosh\left(a+\frac{b}{x}\right)+ib\left(i\sinh(a)\int x\cosh\left(\frac{b}{x}\right)d\frac{1}{x}+i\cosh(a)\int x\sinh\left(\frac{b}{x}\right)d\frac{1}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$x\cosh\left(a+\frac{b}{x}\right)+ib\left(i\sinh(a)\int x\sin\left(\frac{ib}{x}+\frac{\pi}{2}\right)d\frac{1}{x}+i\cosh(a)\int -ix\sin\left(\frac{ib}{x}\right)d\frac{1}{x}\right)$$

$$\Big\downarrow \text{26}$$

$$x\cosh\left(a+\frac{b}{x}\right)+ib\left(i\sinh(a)\int x\sin\left(\frac{ib}{x}+\frac{\pi}{2}\right)d\frac{1}{x}+\cosh(a)\int x\sin\left(\frac{ib}{x}\right)d\frac{1}{x}\right)$$

$$\Big\downarrow \text{3779}$$

$$x\cosh\left(a+\frac{b}{x}\right)+ib\left(i\sinh(a)\int x\sin\left(\frac{ib}{x}+\frac{\pi}{2}\right)d\frac{1}{x}+i\cosh(a)\text{Shi}\left(\frac{b}{x}\right)\right)$$

$$\Big\downarrow \text{3782}$$

$$x\cosh\left(a+\frac{b}{x}\right)+ib\left(i\sinh(a)\text{Chi}\left(\frac{b}{x}\right)+i\cosh(a)\text{Shi}\left(\frac{b}{x}\right)\right)$$

input
```
Int[Cosh[a + b/x],x]
```

output
```
x*Cosh[a + b/x] + I*b*(I*CoshIntegral[b/x]*Sinh[a] + I*Cosh[a]*SinhIntegra
l[b/x])
```

3.25.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5826
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Cosh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

3.25.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

method	result	s
risch	$-\frac{{\mathrm e}^{-a} \operatorname{Ei}_{1}\left(\frac{b}{x}\right) b}{2}+\frac{{\mathrm e}^{-\frac{a x +b}{x}} x}{2}+\frac{{\mathrm e}^{a} \operatorname{Ei}_{1}\left(-\frac{b}{x}\right) b}{2}+\frac{{\mathrm e}^{\frac{a x +b}{x}} x}{2}$	5
meijerg	$-\frac{i \sqrt{\pi}\, \cosh\left(a \right) b \left(\frac{4 i x \cosh\left(\frac{b}{x}\right)}{b \sqrt{\pi}}-\frac{4 i \operatorname{Shi}\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{4}-\frac{\sqrt{\pi}\, \sinh\left(a \right) b \left(\frac{4 \gamma -4-4 \ln\left(x \right)+4 \ln\left(i b \right)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4 x \sinh\left(\frac{b}{x}\right)}{\sqrt{\pi}\, b}+\frac{4 \operatorname{Chi}\left(\frac{b}{x}\right)-4 \ln\left(\frac{b}{x}\right)-4 \gamma}{\sqrt{\pi}}\right)}{4}$	1

input
```
int(cosh(a+b/x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(-a)*Ei(1,b/x)*b+1/2*exp(-(a*x+b)/x)*x+1/2*exp(a)*Ei(1,-b/x)*b+1/2
*exp((a*x+b)/x)*x
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

$$\int \cosh\left(a+\frac{b}{x}\right)\,dx = -\frac{1}{2}\left(b\mathrm{Ei}\left(\frac{b}{x}\right)-b\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\cosh\left(a\right)+x\cosh\left(\frac{ax+b}{x}\right) - \frac{1}{2}\left(b\mathrm{Ei}\left(\frac{b}{x}\right)+b\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\sinh\left(a\right)$$

input
```
integrate(cosh(a+b/x),x, algorithm="fricas")
```

output
```
-1/2*(b*Ei(b/x) - b*Ei(-b/x))*cosh(a) + x*cosh((a*x + b)/x) - 1/2*(b*Ei(b/
x) + b*Ei(-b/x))*sinh(a)
```

3.25.6 Sympy [F]

$$\int \cosh\left(a+\frac{b}{x}\right)\,dx = \int \cosh\left(a+\frac{b}{x}\right)\,dx$$

input
```
integrate(cosh(a+b/x),x)
```

output
```
Integral(cosh(a + b/x), x)
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \cosh\left(a+\frac{b}{x}\right)\,dx = \frac{1}{2}\left(\text{Ei}\left(-\frac{b}{x}\right)e^{(-a)} - \text{Ei}\left(\frac{b}{x}\right)e^{a}\right)b + x\cosh\left(a+\frac{b}{x}\right)$$

input
```
integrate(cosh(a+b/x),x, algorithm="maxima")
```

output
```
1/2*(Ei(-b/x)*e^(-a) - Ei(b/x)*e^a)*b + x*cosh(a + b/x)
```

3.25.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(33) = 66.

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 5.24

$$\int \cosh\left(a+\frac{b}{x}\right)\,dx = \frac{ab^2\text{Ei}\left(a-\frac{ax+b}{x}\right)e^{(-a)} - \frac{(ax+b)b^2\text{Ei}\left(a-\frac{ax+b}{x}\right)e^{(-a)}}{x} - b^2e^{\left(-\frac{ax+b}{x}\right)}}{2\left(a-\frac{ax+b}{x}\right)b} - \frac{ab^2\text{Ei}\left(-a+\frac{ax+b}{x}\right)e^{a} - \frac{(ax+b)b^2\text{Ei}\left(-a+\frac{ax+b}{x}\right)e^{a}}{x} + b^2e^{\left(\frac{ax+b}{x}\right)}}{2\left(a-\frac{ax+b}{x}\right)b}$$

input
```
integrate(cosh(a+b/x),x, algorithm="giac")
```

output
```
1/2*(a*b^2*Ei(a - (a*x + b)/x)*e^(-a) - (a*x + b)*b^2*Ei(a - (a*x + b)/x)*
e^(-a)/x - b^2*e^(-(a*x + b)/x))/((a - (a*x + b)/x)*b) - 1/2*(a*b^2*Ei(-a
+ (a*x + b)/x)*e^a - (a*x + b)*b^2*Ei(-a + (a*x + b)/x)*e^a/x + b^2*e^((a*
x + b)/x))/((a - (a*x + b)/x)*b)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int \cosh\left(a+\frac{b}{x}\right)\,dx=\int \cosh\left(a+\frac{b}{x}\right)\,dx$$

input
```
int(cosh(a + b/x),x)
```

output
```
int(cosh(a + b/x), x)
```

3.26 $\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x} dx$

3.26.1 Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x} dx = -\cosh(a)\text{Chi}\left(\frac{b}{x}\right) - \sinh(a)\text{Shi}\left(\frac{b}{x}\right)$$

output
```
-Chi(b/x)*cosh(a)-Shi(b/x)*sinh(a)
```

3.26.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x} dx = -\cosh(a)\text{Chi}\left(\frac{b}{x}\right) - \sinh(a)\text{Shi}\left(\frac{b}{x}\right)$$

input
```
Integrate[Cosh[a + b/x]/x,x]
```

output
```
-(Cosh[a]*CoshIntegral[b/x]) - Sinh[a]*SinhIntegral[b/x]
```

3.26.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5842, 5839, 5840}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x}\,dx$$

$$\Big\downarrow \text{ 5842}$$

$$\sinh(a)\int \frac{\sinh\left(\frac{b}{x}\right)}{x}dx+\cosh(a)\int \frac{\cosh\left(\frac{b}{x}\right)}{x}dx$$

$$\Big\downarrow \text{ 5839}$$

$$\cosh(a)\int \frac{\cosh\left(\frac{b}{x}\right)}{x}dx-\sinh(a)\text{Shi}\left(\frac{b}{x}\right)$$

$$\Big\downarrow \text{ 5840}$$

$$-\cosh(a)\text{Chi}\left(\frac{b}{x}\right)-\sinh(a)\text{Shi}\left(\frac{b}{x}\right)$$

input
```
Int[Cosh[a + b/x]/x,x]
```

output
```
-(Cosh[a]*CoshIntegral[b/x]) - Sinh[a]*SinhIntegral[b/x]
```

3.26.3.1 Defintions of rubi rules used

rule 5839
```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

rule 5840
```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

rule 5842
```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[Cosh[c]   Int[Cosh
[d*x^n]/x, x], x] + Simp[Sinh[c]   Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d
, n}, x]
```

3.26.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

method	result	size
risch	$\frac{e^{-a} \operatorname{Ei}_1\left(\frac{b}{x}\right)}{2}+\frac{e^{a} \operatorname{Ei}_1\left(-\frac{b}{x}\right)}{2}$	27
meijerg	$-\frac{\sqrt{\pi}\,\cosh(a)\left(\frac{2\gamma-2\ln(x)+2\ln(ib)}{\sqrt{\pi}}+\frac{2\,\operatorname{Chi}\left(\frac{b}{x}\right)-2\ln\left(\frac{b}{x}\right)-2\gamma}{\sqrt{\pi}}\right)}{2}-\operatorname{Shi}\left(\frac{b}{x}\right)\sinh(a)$	62

input
```
int(cosh(a+b/x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(-a)*Ei(1,b/x)+1/2*exp(a)*Ei(1,-b/x)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x}\,dx=-\frac{1}{2}\left(\operatorname{Ei}\left(\frac{b}{x}\right)+\operatorname{Ei}\left(-\frac{b}{x}\right)\right)\cosh(a)-\frac{1}{2}\left(\operatorname{Ei}\left(\frac{b}{x}\right)-\operatorname{Ei}\left(-\frac{b}{x}\right)\right)\sinh(a)$$

input
```
integrate(cosh(a+b/x)/x,x, algorithm="fricas")
```

output
```
-1/2*(Ei(b/x) + Ei(-b/x))*cosh(a) - 1/2*(Ei(b/x) - Ei(-b/x))*sinh(a)
```

3.26.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x}\,dx=-\sinh(a)\operatorname{Shi}\left(\frac{b}{x}\right)-\cosh(a)\operatorname{Chi}\left(\frac{b}{x}\right)$$

input
```
integrate(cosh(a+b/x)/x,x)
```

output
```
-sinh(a)*Shi(b/x) - cosh(a)*Chi(b/x)
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x}\,dx = -\frac{1}{2}\,\mathrm{Ei}\left(-\frac{b}{x}\right)e^{(-a)} - \frac{1}{2}\,\mathrm{Ei}\left(\frac{b}{x}\right)e^{a}$$

input
```
integrate(cosh(a+b/x)/x,x, algorithm="maxima")
```

output
```
-1/2*Ei(-b/x)*e^(-a) - 1/2*Ei(b/x)*e^a
```

3.26.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(21) = 42.

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.05

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x}\,dx = -\frac{b\mathrm{Ei}\left(a-\frac{ax+b}{x}\right)e^{(-a)} + b\mathrm{Ei}\left(-a+\frac{ax+b}{x}\right)e^{a}}{2\,b}$$

input
```
integrate(cosh(a+b/x)/x,x, algorithm="giac")
```

output
```
-1/2*(b*Ei(a - (a*x + b)/x)*e^(-a) + b*Ei(-a + (a*x + b)/x)*e^a)/b
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x}\,dx = -\cosh(a)\,\mathrm{coshint}\left(\frac{b}{x}\right) - \sinh(a)\,\mathrm{sinhint}\left(\frac{b}{x}\right)$$

input
```
int(cosh(a + b/x)/x,x)
```

output
```
- cosh(a)*coshint(b/x) - sinh(a)*sinhint(b/x)
```

3.27 $\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^2}\,dx$

3.27.1 Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{\sinh\left(a+\frac{b}{x}\right)}{b}$$

output
```
-sinh(a+b/x)/b
```

3.27.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{\sinh\left(a+\frac{b}{x}\right)}{b}$$

input
```
Integrate[Cosh[a + b/x]/x^2,x]
```

output
```
-(Sinh[a + b/x]/b)
```

3.27.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5844, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^2}dx$$

$$\Big\downarrow \text{5844}$$

$$-\int \cosh\left(a+\frac{b}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int \sin\left(ia+\frac{ib}{x}+\frac{\pi}{2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{\sinh\left(a+\frac{b}{x}\right)}{b}$$

input
```
Int[Cosh[a + b/x]/x^2,x]
```

output
```
-(Sinh[a + b/x]/b)
```

3.27.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

3.27.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\sinh\left(a+\frac{b}{x}\right)}{b}$	14
default	$-\frac{\sinh\left(a+\frac{b}{x}\right)}{b}$	14
parallelrisch	$-\frac{\sinh\left(\frac{ax+b}{x}\right)}{b}$	16
risch	$-\frac{\mathrm{e}^{\frac{ax+b}{x}}}{2b}+\frac{\mathrm{e}^{-\frac{ax+b}{x}}}{2b}$	33
meijerg	$-\frac{\cosh(a)\sinh\left(\frac{b}{x}\right)}{b}+\frac{\sqrt{\pi}\,\sinh(a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{b}$	39

input
```
int(cosh(a+b/x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-sinh(a+b/x)/b
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^2}\,dx=-\frac{\sinh\left(\frac{ax+b}{x}\right)}{b}$$

input
```
integrate(cosh(a+b/x)/x^2,x, algorithm="fricas")
```

output
```
-sinh((a*x + b)/x)/b
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^2}\,dx = \begin{cases} -\frac{\sinh\left(a+\frac{b}{x}\right)}{b} & \text{for } b \neq 0 \\ -\frac{\cosh(a)}{x} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(a+b/x)/x**2,x)
```

output
```
Piecewise((-sinh(a + b/x)/b, Ne(b, 0)), (-cosh(a)/x, True))
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{\sinh\left(a+\frac{b}{x}\right)}{b}$$

input
```
integrate(cosh(a+b/x)/x^2,x, algorithm="maxima")
```

output
```
-sinh(a + b/x)/b
```

3.27.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.23

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{e^{\left(\frac{ax+b}{x}\right)} - e^{\left(-\frac{ax+b}{x}\right)}}{2\,b}$$

input
```
integrate(cosh(a+b/x)/x^2,x, algorithm="giac")
```

output
```
-1/2*(e^((a*x + b)/x) - e^(-(a*x + b)/x))/b
```

3.27.9 Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{\sinh\left(a+\frac{b}{x}\right)}{b}$$

input
```
int(cosh(a + b/x)/x^2,x)
```

output
```
-sinh(a + b/x)/b
```

3.28 $\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^3}\,dx$

3.28.1 Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^3}\,dx = \frac{\cosh\left(a+\frac{b}{x}\right)}{b^2} - \frac{\sinh\left(a+\frac{b}{x}\right)}{bx}$$

output
```
cosh(a+b/x)/b^2-sinh(a+b/x)/b/x
```

3.28.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^3}\,dx = \frac{x\cosh\left(a+\frac{b}{x}\right) - b\sinh\left(a+\frac{b}{x}\right)}{b^2x}$$

input
```
Integrate[Cosh[a + b/x]/x^3,x]
```

output
```
(x*Cosh[a + b/x] - b*Sinh[a + b/x])/(b^2*x)
```

3.28.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5844, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh \left(a+\frac{b}{x}\right)}{x^3} \, dx$$

$$\Big\downarrow \text{5844}$$

$$-\int \frac{\cosh \left(a+\frac{b}{x}\right)}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\sin \left(i a+\frac{i b}{x}+\frac{\pi}{2}\right)}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{\sinh \left(a+\frac{b}{x}\right)}{b x}+\frac{i \int -i \sinh \left(a+\frac{b}{x}\right)d\frac{1}{x}}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \sinh \left(a+\frac{b}{x}\right)d\frac{1}{x}}{b}-\frac{\sinh \left(a+\frac{b}{x}\right)}{b x}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh \left(a+\frac{b}{x}\right)}{b x}+\frac{\int -i \sin \left(i a+\frac{i b}{x}\right)d\frac{1}{x}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\sinh \left(a+\frac{b}{x}\right)}{b x}-\frac{i \int \sin \left(i a+\frac{i b}{x}\right)d\frac{1}{x}}{b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\cosh \left(a+\frac{b}{x}\right)}{b^2}-\frac{\sinh \left(a+\frac{b}{x}\right)}{b x}$$

input
```
Int[Cosh[a + b/x]/x^3,x]
```

output
```
Cosh[a + b/x]/b^2 - Sinh[a + b/x]/(b*x)
```

3.28. $\int \frac{\cosh \left(a+\frac{b}{x}\right)}{x^3} \, dx$

3.28.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

3.28.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

method	result	size
parallelrisch	$\frac{2\tanh\left(\frac{ax+b}{2x}\right)b-2x}{x\,b^{2}\left(\tanh\left(\frac{ax+b}{2x}\right)^{2}-1\right)}$	43
derivativedivides	$-\frac{\left(a+\frac{b}{x}\right)\sinh\left(a+\frac{b}{x}\right)-\cosh\left(a+\frac{b}{x}\right)-a\sinh\left(a+\frac{b}{x}\right)}{b^{2}}$	44
default	$-\frac{\left(a+\frac{b}{x}\right)\sinh\left(a+\frac{b}{x}\right)-\cosh\left(a+\frac{b}{x}\right)-a\sinh\left(a+\frac{b}{x}\right)}{b^{2}}$	44
risch	$-\frac{(-x+b)\,\mathrm{e}^{\frac{ax+b}{x}}}{2b^{2}x}+\frac{(x+b)\,\mathrm{e}^{-\frac{ax+b}{x}}}{2b^{2}x}$	47
meijerg	$\frac{2\sqrt{\pi}\,\cosh(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh\left(\frac{b}{x}\right)}{2\sqrt{\pi}}-\frac{b\sinh\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}\right)}{b^{2}}-\frac{\sinh(a)\left(\frac{\cosh\left(\frac{b}{x}\right)b}{x}-\sinh\left(\frac{b}{x}\right)\right)}{b^{2}}$	71

input
```
int(cosh(a+b/x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
(2*tanh(1/2*(a*x+b)/x)*b-2*x)/x/b^2/(tanh(1/2*(a*x+b)/x)^2-1)
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^3}\,dx = \frac{x\cosh\left(\frac{ax+b}{x}\right)-b\sinh\left(\frac{ax+b}{x}\right)}{b^2x}$$

input
```
integrate(cosh(a+b/x)/x^3,x, algorithm="fricas")
```

output
```
(x*cosh((a*x + b)/x) - b*sinh((a*x + b)/x))/(b^2*x)
```

3.28.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^3}\,dx = \begin{cases} -\frac{\sinh\left(a+\frac{b}{x}\right)}{bx}+\frac{\cosh\left(a+\frac{b}{x}\right)}{b^2} & \text{for } b\neq 0\\ -\frac{\cosh(a)}{2x^2} & \text{otherwise}\end{cases}$$

input
```
integrate(cosh(a+b/x)/x**3,x)
```

output
```
Piecewise((-sinh(a + b/x)/(b*x) + cosh(a + b/x)/b**2, Ne(b, 0)), (-cosh(a)
/(2*x**2), True))
```

3.28.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^3}\,dx = \frac{1}{4}\,b\left(\frac{e^{(-a)}\Gamma\left(3,\frac{b}{x}\right)}{b^3}+\frac{e^{a}\Gamma\left(3,-\frac{b}{x}\right)}{b^3}\right)-\frac{\cosh\left(a+\frac{b}{x}\right)}{2\,x^2}$$

input
```
integrate(cosh(a+b/x)/x^3,x, algorithm="maxima")
```

output
```
1/4*b*(e^(-a)*gamma(3, b/x)/b^3 + e^a*gamma(3, -b/x)/b^3) - 1/2*cosh(a + b
/x)/x^2
```

3.28.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(29) = 58.

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.21

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^3}\,dx$$
$$= \frac{ae^{\left(\frac{ax+b}{x}\right)}-ae^{\left(-\frac{ax+b}{x}\right)}-\frac{(ax+b)e^{\left(\frac{ax+b}{x}\right)}}{x}+\frac{(ax+b)e^{\left(-\frac{ax+b}{x}\right)}}{x}+e^{\left(\frac{ax+b}{x}\right)}+e^{\left(-\frac{ax+b}{x}\right)}}{2\,b^2}$$

input
```
integrate(cosh(a+b/x)/x^3,x, algorithm="giac")
```

output
```
1/2*(a*e^((a*x + b)/x) - a*e^(-(a*x + b)/x) - (a*x + b)*e^((a*x + b)/x)/x
+ (a*x + b)*e^(-(a*x + b)/x)/x + e^((a*x + b)/x) + e^(-(a*x + b)/x))/b^2
```

3.28.9 Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^3}\,dx = \frac{\cosh\left(a+\frac{b}{x}\right)}{b^2}-\frac{\sinh\left(a+\frac{b}{x}\right)}{b\,x}$$

input
```
int(cosh(a + b/x)/x^3,x)
```

output
```
cosh(a + b/x)/b^2 - sinh(a + b/x)/(b*x)
```

3.29 $\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^4}\,dx$

3.29.1 Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^4}\,dx=\frac{2\cosh\left(a+\frac{b}{x}\right)}{b^2x}-\frac{2\sinh\left(a+\frac{b}{x}\right)}{b^3}-\frac{\sinh\left(a+\frac{b}{x}\right)}{bx^2}$$

output
```
2*cosh(a+b/x)/b^2/x-2*sinh(a+b/x)/b^3-sinh(a+b/x)/b/x^2
```

3.29.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^4}\,dx=\frac{2bx\cosh\left(a+\frac{b}{x}\right)-(b^2+2x^2)\sinh\left(a+\frac{b}{x}\right)}{b^3x^2}$$

input
```
Integrate[Cosh[a + b/x]/x^4,x]
```

output
```
(2*b*x*Cosh[a + b/x] - (b^2 + 2*x^2)*Sinh[a + b/x])/(b^3*x^2)
```

3.29.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5844, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^4}dx$$

$$\Big\downarrow \text{ 5844}$$

$$-\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 3042}$$

$$-\int \frac{\sin\left(ia+\frac{ib}{x}+\frac{\pi}{2}\right)}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{\sinh\left(a+\frac{b}{x}\right)}{bx^2}+\frac{2i\int -\frac{i\sinh\left(a+\frac{b}{x}\right)}{x}d\frac{1}{x}}{b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{2\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x}d\frac{1}{x}}{b}-\frac{\sinh\left(a+\frac{b}{x}\right)}{bx^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\sinh\left(a+\frac{b}{x}\right)}{bx^2}+\frac{2\int -\frac{i\sin\left(ia+\frac{ib}{x}\right)}{x}d\frac{1}{x}}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{\sinh\left(a+\frac{b}{x}\right)}{bx^2}-\frac{2i\int \frac{\sin\left(ia+\frac{ib}{x}\right)}{x}d\frac{1}{x}}{b}$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{\sinh\left(a+\frac{b}{x}\right)}{bx^2}-\frac{2i\left(\frac{i\cosh\left(a+\frac{b}{x}\right)}{bx}-\frac{i\int \cosh\left(a+\frac{b}{x}\right)d\frac{1}{x}}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh\left(a+\frac{b}{x}\right)}{bx^2}-\frac{2i\left(\frac{i\cosh\left(a+\frac{b}{x}\right)}{bx}-\frac{i\int\sin\left(ia+\frac{ib}{x}+\frac{\pi}{2}\right)d\frac{1}{x}}{b}\right)}{b}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{\sinh\left(a+\frac{b}{x}\right)}{bx^2}-\frac{2i\left(\frac{i\cosh\left(a+\frac{b}{x}\right)}{bx}-\frac{i\sinh\left(a+\frac{b}{x}\right)}{b^2}\right)}{b}$$

input
```
Int[Cosh[a + b/x]/x^4,x]
```

output
```
-(Sinh[a + b/x]/(b*x^2)) - ((2*I)*((I*Cosh[a + b/x])/(b*x) - (I*Sinh[a + b
/x])/b^2))/b
```

3.29.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

3.29.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{(b^2-2bx+2x^2)\,\mathrm{e}^{\frac{ax+b}{x}}}{2b^3x^2}+\frac{(b^2+2bx+2x^2)\,\mathrm{e}^{-\frac{ax+b}{x}}}{2b^3x^2}$
parallelrisch	$\frac{-2\tanh\left(\frac{ax+b}{2x}\right)^2 xb+4\tanh\left(\frac{ax+b}{2x}\right)x^2+2\tanh\left(\frac{ax+b}{2x}\right)b^2-2bx}{x^2b^3\left(\tanh\left(\frac{ax+b}{2x}\right)^2-1\right)}$
derivativedivides	$-\frac{a^2\sinh\left(a+\frac{b}{x}\right)-2a\left(\left(a+\frac{b}{x}\right)\sinh\left(a+\frac{b}{x}\right)-\cosh\left(a+\frac{b}{x}\right)\right)+\left(a+\frac{b}{x}\right)^2\sinh\left(a+\frac{b}{x}\right)-2\left(a+\frac{b}{x}\right)\cosh\left(a+\frac{b}{x}\right)+2\sinh\left(a+\frac{b}{x}\right.}{b^3}$
default	$-\frac{a^2\sinh\left(a+\frac{b}{x}\right)-2a\left(\left(a+\frac{b}{x}\right)\sinh\left(a+\frac{b}{x}\right)-\cosh\left(a+\frac{b}{x}\right)\right)+\left(a+\frac{b}{x}\right)^2\sinh\left(a+\frac{b}{x}\right)-2\left(a+\frac{b}{x}\right)\cosh\left(a+\frac{b}{x}\right)+2\sinh\left(a+\frac{b}{x}\right.}{b^3}$
meijerg	$-\frac{4i\sqrt{\pi}\,\cosh(a)\left(\frac{ib\cosh\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}-\frac{i\left(\frac{3b^2}{2x^2}+3\right)\sinh\left(\frac{b}{x}\right)}{6\sqrt{\pi}}\right)}{b^3}-\frac{4\sqrt{\pi}\,\sinh(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{b^2}{2x^2}+1\right)\cosh\left(\frac{b}{x}\right)}{2\sqrt{\pi}}-\frac{b\sinh\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}\right)}{b^3}$

input
```
int(cosh(a+b/x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b^2-2*b*x+2*x^2)/b^3/x^2*exp((a*x+b)/x)+1/2*(b^2+2*b*x+2*x^2)/b^3/x^
2*exp(-(a*x+b)/x)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^4}\,dx = \frac{2\,bx\cosh\left(\frac{ax+b}{x}\right)-(b^2+2\,x^2)\sinh\left(\frac{ax+b}{x}\right)}{b^3x^2}$$

input
```
integrate(cosh(a+b/x)/x^4,x, algorithm="fricas")
```

output
```
(2*b*x*cosh((a*x + b)/x) - (b^2 + 2*x^2)*sinh((a*x + b)/x))/(b^3*x^2)
```

3.29.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^4}\,dx = \begin{cases} -\frac{\sinh\left(a+\frac{b}{x}\right)}{bx^2} + \frac{2\cosh\left(a+\frac{b}{x}\right)}{b^2x} - \frac{2\sinh\left(a+\frac{b}{x}\right)}{b^3} & \text{for } b \neq 0 \\ -\frac{\cosh(a)}{3x^3} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(a+b/x)/x**4,x)
```

output
```
Piecewise((-sinh(a + b/x)/(b*x**2) + 2*cosh(a + b/x)/(b**2*x) - 2*sinh(a +
 b/x)/b**3, Ne(b, 0)), (-cosh(a)/(3*x**3), True))
```

3.29.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^4}\,dx = \frac{1}{6}\,b\left(\frac{e^{(-a)}\Gamma\left(4,\frac{b}{x}\right)}{b^4} - \frac{e^{a}\Gamma\left(4,-\frac{b}{x}\right)}{b^4}\right) - \frac{\cosh\left(a+\frac{b}{x}\right)}{3\,x^3}$$

input
```
integrate(cosh(a+b/x)/x^4,x, algorithm="maxima")
```

output
```
1/6*b*(e^(-a)*gamma(4, b/x)/b^4 - e^a*gamma(4, -b/x)/b^4) - 1/3*cosh(a + b
/x)/x^3
```

3.29.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(46) = 92.

Time = 0.31 (sec) , antiderivative size = 216, normalized size of antiderivative = 4.70

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^4}\,dx =$$

$$-\frac{a^2e^{\left(\frac{ax+b}{x}\right)} - a^2e^{\left(-\frac{ax+b}{x}\right)} + 2\,ae^{\left(\frac{ax+b}{x}\right)} - \frac{2\,(ax+b)ae^{\left(\frac{ax+b}{x}\right)}}{x} + 2\,ae^{\left(-\frac{ax+b}{x}\right)} + \frac{2\,(ax+b)ae^{\left(-\frac{ax+b}{x}\right)}}{x} + \frac{(ax+b)^2e^{\left(\frac{ax+b}{x}\right)}}{x^2}}{2\,b^3}$$

input
```
integrate(cosh(a+b/x)/x^4,x, algorithm="giac")
```

output
```
-1/2*(a^2*e^((a*x + b)/x) - a^2*e^(-(a*x + b)/x) + 2*a*e^((a*x + b)/x) - 2
*(a*x + b)*a*e^((a*x + b)/x)/x + 2*a*e^(-(a*x + b)/x) + 2*(a*x + b)*a*e^(-
(a*x + b)/x)/x + (a*x + b)^2*e^((a*x + b)/x)/x^2 - 2*(a*x + b)*e^((a*x + b
)/x)/x - (a*x + b)^2*e^(-(a*x + b)/x)/x^2 - 2*(a*x + b)*e^(-(a*x + b)/x)/x
 + 2*e^((a*x + b)/x) - 2*e^(-(a*x + b)/x))/b^3
```

3.29.9 Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^4}\,dx = \frac{e^{-a-\frac{b}{x}}\left(\frac{x}{b^2}+\frac{1}{2b}+\frac{x^2}{b^3}\right)}{x^2} - \frac{e^{a+\frac{b}{x}}\left(\frac{1}{2b}-\frac{x}{b^2}+\frac{x^2}{b^3}\right)}{x^2}$$

input
```
int(cosh(a + b/x)/x^4,x)
```

output
```
(exp(- a - b/x)*(x/b^2 + 1/(2*b) + x^2/b^3))/x^2 - (exp(a + b/x)*(1/(2*b)
- x/b^2 + x^2/b^3))/x^2
```

3.30 $\int \cosh\left(a+\frac{b}{x^2}\right)\,dx$

3.30.1 Optimal result

Integrand size = 8, antiderivative size = 67

$$\int \cosh\left(a+\frac{b}{x^2}\right)\,dx = x\cosh\left(a+\frac{b}{x^2}\right)+\frac{1}{2}\sqrt{b}e^{-a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{1}{2}\sqrt{b}e^{a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)$$

output
```
x*cosh(a+b/x^2)+1/2*erf(b^(1/2)/x)*b^(1/2)*Pi^(1/2)/exp(a)-1/2*exp(a)*erfi
(b^(1/2)/x)*b^(1/2)*Pi^(1/2)
```

3.30.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int \cosh\left(a+\frac{b}{x^2}\right)\,dx = x\cosh\left(a+\frac{b}{x^2}\right)+\frac{1}{2}\sqrt{b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)-\sinh(a)) - \frac{1}{2}\sqrt{b}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)+\sinh(a))$$

input
```
Integrate[Cosh[a + b/x^2],x]
```

output
```
x*Cosh[a + b/x^2] + (Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a]))/
2 - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[b]/x]*(Cosh[a] + Sinh[a]))/2
```

3.30.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5826, 5850, 5821, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh\left(a+\frac{b}{x^2}\right)dx$$

$$\Big\downarrow \text{ 5826}$$

$$-\int x^2\cosh\left(a+\frac{b}{x^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{ 5850}$$

$$x\cosh\left(a+\frac{b}{x^2}\right)-2b\int \sinh\left(a+\frac{b}{x^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{ 5821}$$

$$x\cosh\left(a+\frac{b}{x^2}\right)-2b\left(\frac{1}{2}\int e^{a+\frac{b}{x^2}}d\frac{1}{x}-\frac{1}{2}\int e^{-a-\frac{b}{x^2}}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 2633}$$

$$x\cosh\left(a+\frac{b}{x^2}\right)-2b\left(\frac{\sqrt{\pi}e^a\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}-\frac{1}{2}\int e^{-a-\frac{b}{x^2}}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 2634}$$

$$x\cosh\left(a+\frac{b}{x^2}\right)-2b\left(\frac{\sqrt{\pi}e^a\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}e^{-a}\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}\right)$$

input
```
Int[Cosh[a + b/x^2],x]
```

output
```
x*Cosh[a + b/x^2] - 2*b*(-1/4*(Sqrt[Pi]*Erf[Sqrt[b]/x])/(Sqrt[b]*E^a) + (E
^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/(4*Sqrt[b]))
```

3.30.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 5821
```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[1/2   Int[E^(c + d*x^n
), x], x] - Simp[1/2   Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IG
tQ[n, 1]
```

rule 5826
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Cosh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

rule 5850
```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x
)^(m + 1)*(Cosh[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   In
t[(e*x)^(m + n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

3.30.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

method	result
risch	$\frac{\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)\sqrt{b}\,\sqrt{\pi}\,\mathrm{e}^{-a}}{2}+\frac{\mathrm{e}^{-a} x\,\mathrm{e}^{-\frac{b}{x^{2}}}}{2}+\frac{\mathrm{e}^{a}\mathrm{e}^{\frac{b}{x^{2}}} x}{2}-\frac{\mathrm{e}^{a} b\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-b}}{x}\right)}{2\sqrt{-b}}$
meijerg	$-\frac{\sqrt{\pi}\,\cosh(a)\sqrt{2}\,\sqrt{ib}\left(-\frac{2x\sqrt{2}\,\mathrm{e}^{\frac{b}{x^{2}}}}{\sqrt{\pi}\,\sqrt{ib}}-\frac{2x\sqrt{2}\,\mathrm{e}^{-\frac{b}{x^{2}}}}{\sqrt{\pi}\,\sqrt{ib}}-\frac{2\sqrt{2}\,\sqrt{b}\,\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{\sqrt{ib}}+\frac{2\sqrt{2}\,\sqrt{b}\,\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{\sqrt{ib}}\right)}{8}+\frac{i\sqrt{\pi}\,\sinh(a)\sqrt{2}\,\sqrt{ib}\left(\frac{2x\sqrt{2}\,\sqrt{ib}\,\mathrm{e}^{-\frac{b}{x^{2}}}}{\sqrt{\pi}\,b}-\right.}{}$

input
```
int(cosh(a+b/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*erf(b^(1/2)/x)*b^(1/2)*Pi^(1/2)/exp(a)+1/2/exp(a)*x*exp(-b/x^2)+1/2*ex
p(a)*exp(b/x^2)*x-1/2*exp(a)*b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(49) = 98$.

Time = 0.26 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.36

$$\int \cosh\left(a+\frac{b}{x^2}\right)\,dx = \frac{x\cosh\left(\frac{ax^2+b}{x^2}\right)^2+\sqrt{\pi}\left(\cosh(a)\cosh\left(\frac{ax^2+b}{x^2}\right)+\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh(a)+(\cosh(a)+\sinh(a))\sinh\left(\frac{ax^2+b}{x^2}\right)\right.}{}$$

input
```
integrate(cosh(a+b/x^2),x, algorithm="fricas")
```

output
```
1/2*(x*cosh((a*x^2 + b)/x^2)^2 + sqrt(pi)*(cosh(a)*cosh((a*x^2 + b)/x^2) +
 cosh((a*x^2 + b)/x^2)*sinh(a) + (cosh(a) + sinh(a))*sinh((a*x^2 + b)/x^2)
)*sqrt(-b)*erf(sqrt(-b)/x) + sqrt(pi)*(cosh(a)*cosh((a*x^2 + b)/x^2) - cos
h((a*x^2 + b)/x^2)*sinh(a) + (cosh(a) - sinh(a))*sinh((a*x^2 + b)/x^2))*sq
rt(b)*erf(sqrt(b)/x) + 2*x*cosh((a*x^2 + b)/x^2)*sinh((a*x^2 + b)/x^2) + x
*sinh((a*x^2 + b)/x^2)^2 + x)/(cosh((a*x^2 + b)/x^2) + sinh((a*x^2 + b)/x^
2))
```

3.30.6 Sympy [F]

$$\int \cosh\left(a+\frac{b}{x^2}\right)\,dx = \int \cosh\left(a+\frac{b}{x^2}\right)\,dx$$

input
```
integrate(cosh(a+b/x**2),x)
```

output
```
Integral(cosh(a + b/x**2), x)
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

$$\int \cosh \left(a+\frac{b}{x^2}\right) dx = \frac{1}{2} b \left(\frac{\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{\frac{b}{x^2}}\right)-1\right)e^{(-a)}}{x\sqrt{\frac{b}{x^2}}} - \frac{\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-\frac{b}{x^2}}\right)-1\right)e^{a}}{x\sqrt{-\frac{b}{x^2}}}\right) + x\cosh\left(a+\frac{b}{x^2}\right)$$

input
```
integrate(cosh(a+b/x^2),x, algorithm="maxima")
```

output
```
1/2*b*(sqrt(pi)*(erf(sqrt(b/x^2)) - 1)*e^(-a)/(x*sqrt(b/x^2)) - sqrt(pi)*(
erf(sqrt(-b/x^2)) - 1)*e^a/(x*sqrt(-b/x^2))) + x*cosh(a + b/x^2)
```

3.30.8 Giac [F]

$$\int \cosh \left(a+\frac{b}{x^2}\right) dx = \int \cosh \left(a+\frac{b}{x^2}\right) dx$$

input
```
integrate(cosh(a+b/x^2),x, algorithm="giac")
```

output
```
integrate(cosh(a + b/x^2), x)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \cosh \left(a+\frac{b}{x^2}\right) dx = \int \cosh\left(a+\frac{b}{x^2}\right) dx$$

input
```
int(cosh(a + b/x^2),x)
```

output
```
int(cosh(a + b/x^2), x)
```

3.31 $\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x}\,dx$

3.31.1 Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x}\,dx = -\frac{1}{2}\cosh(a)\text{Chi}\left(\frac{b}{x^2}\right) - \frac{1}{2}\sinh(a)\text{Shi}\left(\frac{b}{x^2}\right)$$

output
```
-1/2*Chi(b/x^2)*cosh(a)-1/2*Shi(b/x^2)*sinh(a)
```

3.31.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x}\,dx = \frac{1}{2}\left(-\cosh(a)\text{Chi}\left(\frac{b}{x^2}\right) - \sinh(a)\text{Shi}\left(\frac{b}{x^2}\right)\right)$$

input
```
Integrate[Cosh[a + b/x^2]/x,x]
```

output
```
(-(Cosh[a]*CoshIntegral[b/x^2]) - Sinh[a]*SinhIntegral[b/x^2])/2
```

3.31.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5842, 5839, 5840}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x}dx$$

$$\Big\downarrow \text{5842}$$

$$\sinh(a)\int \frac{\sinh\left(\frac{b}{x^2}\right)}{x}dx+\cosh(a)\int \frac{\cosh\left(\frac{b}{x^2}\right)}{x}dx$$

$$\Big\downarrow \text{5839}$$

$$\cosh(a)\int \frac{\cosh\left(\frac{b}{x^2}\right)}{x}dx-\frac{1}{2}\sinh(a)\text{Shi}\left(\frac{b}{x^2}\right)$$

$$\Big\downarrow \text{5840}$$

$$-\frac{1}{2}\cosh(a)\text{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{2}\sinh(a)\text{Shi}\left(\frac{b}{x^2}\right)$$

input
```
Int[Cosh[a + b/x^2]/x,x]
```

output
```
-1/2*(Cosh[a]*CoshIntegral[b/x^2]) - (Sinh[a]*SinhIntegral[b/x^2])/2
```

3.31.3.1 Defintions of rubi rules used

rule 5839
```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

rule 5840
```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

rule 5842
```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[Cosh[c]   Int[Cosh
[d*x^n]/x, x], x] + Simp[Sinh[c]   Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d
, n}, x]
```

3.31.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

method	result	size
risch	$\frac{e^{2a} e^{-a} \operatorname{Ei}_1\left(-\frac{b}{x^2}\right)}{4}+\frac{e^{-a} \operatorname{Ei}_1\left(\frac{b}{x^2}\right)}{4}$	33
meijerg	$-\frac{\sqrt{\pi}\, \cosh(a)\left(\frac{2\gamma-4\ln(x)+2\ln(ib)}{\sqrt{\pi}}+\frac{2\operatorname{Chi}\left(\frac{b}{x^2}\right)-2\ln\left(\frac{b}{x^2}\right)-2\gamma}{\sqrt{\pi}}\right)}{4}-\frac{\operatorname{Shi}\left(\frac{b}{x^2}\right)\sinh(a)}{2}$	62

input
```
int(cosh(a+b/x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(2*a)*exp(-a)*Ei(1,-b/x^2)+1/4*exp(-a)*Ei(1,b/x^2)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x}\,dx = -\frac{1}{4}\left(\operatorname{Ei}\left(\frac{b}{x^2}\right)+\operatorname{Ei}\left(-\frac{b}{x^2}\right)\right)\cosh(a) - \frac{1}{4}\left(\operatorname{Ei}\left(\frac{b}{x^2}\right)-\operatorname{Ei}\left(-\frac{b}{x^2}\right)\right)\sinh(a)$$

input
```
integrate(cosh(a+b/x^2)/x,x, algorithm="fricas")
```

output
```
-1/4*(Ei(b/x^2) + Ei(-b/x^2))*cosh(a) - 1/4*(Ei(b/x^2) - Ei(-b/x^2))*sinh(
a)
```

3.31.6 Sympy [F]

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x}\,dx = \int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x}\,dx$$

input
```
integrate(cosh(a+b/x**2)/x,x)
```

output
```
Integral(cosh(a + b/x**2)/x, x)
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x}\,dx = -\frac{1}{4}\,\mathrm{Ei}\left(-\frac{b}{x^2}\right)e^{(-a)} - \frac{1}{4}\,\mathrm{Ei}\left(\frac{b}{x^2}\right)e^{a}$$

input
```
integrate(cosh(a+b/x^2)/x,x, algorithm="maxima")
```

output
```
-1/4*Ei(-b/x^2)*e^(-a) - 1/4*Ei(b/x^2)*e^a
```

3.31.8 Giac [F]

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x}\,dx = \int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x}\,dx$$

input
```
integrate(cosh(a+b/x^2)/x,x, algorithm="giac")
```

output
```
integrate(cosh(a + b/x^2)/x, x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x}\,dx = -\frac{\cosh(a)\,\mathrm{coshint}\left(\frac{b}{x^2}\right)}{2} - \frac{\sinh(a)\,\mathrm{sinhint}\left(\frac{b}{x^2}\right)}{2}$$

input
```
int(cosh(a + b/x^2)/x,x)
```

output
```
- (cosh(a)*coshint(b/x^2))/2 - (sinh(a)*sinhint(b/x^2))/2
```

3.32 $\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$

3.32.1 Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx=-\frac{e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}-\frac{e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}$$

output
```
-1/4*erf(b^(1/2)/x)*Pi^(1/2)/exp(a)/b^(1/2)-1/4*exp(a)*erfi(b^(1/2)/x)*Pi^
(1/2)/b^(1/2)
```

3.32.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx=-\frac{\sqrt{\pi}\left(\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)-\sinh(a))+\mathrm{erfi}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)+\sinh(a))\right)}{4\sqrt{b}}$$

input
```
Integrate[Cosh[a + b/x^2]/x^2,x]
```

output
```
-1/4*(Sqrt[Pi]*(Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a]) + Erfi[Sqrt[b]/x]*(Cosh
[a] + Sinh[a])))/Sqrt[b]
```

3.32.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5870, 5822, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^2}dx$$

$$\Big\downarrow \text{5870}$$

$$-\int \cosh\left(a+\frac{b}{x^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{5822}$$

$$-\frac{1}{2}\int e^{-a-\frac{b}{x^2}}d\frac{1}{x}-\frac{1}{2}\int e^{a+\frac{b}{x^2}}d\frac{1}{x}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{1}{2}\int e^{-a-\frac{b}{x^2}}d\frac{1}{x}-\frac{\sqrt{\pi}e^a\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{\sqrt{\pi}e^{-a}\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}e^a\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}$$

input
```
Int[Cosh[a + b/x^2]/x^2,x]
```

output
```
-1/4*(Sqrt[Pi]*Erf[Sqrt[b]/x])/(Sqrt[b]*E^a) - (E^a*Sqrt[Pi]*Erfi[Sqrt[b]/
x])/(4*Sqrt[b])
```

3.32.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 5822
```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[1/2   Int[E^(c + d*x^n
), x], x] + Simp[1/2   Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IG
tQ[n, 1]
```

rule 5870
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> -Subst[Int[(a + b*Cosh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

3.32.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result	si
risch	$-\frac{\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)\sqrt{\pi}\,\mathrm{e}^{-a}}{4\sqrt{b}}-\frac{\mathrm{e}^{a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-b}}{x}\right)}{4\sqrt{-b}}$	4
meijerg	$\frac{i\sqrt{\pi}\,\cosh(a)\sqrt{2}\,\sqrt{ib}\left(\frac{\sqrt{ib}\,\sqrt{2}\,\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)}{2\sqrt{b}}+\frac{\sqrt{ib}\,\sqrt{2}\,\mathrm{erfi}\left(\frac{\sqrt{b}}{x}\right)}{2\sqrt{b}}\right)}{4b}+\frac{\sqrt{\pi}\,\sinh(a)\sqrt{2}\,\sqrt{ib}\left(-\frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)}{2b^{\frac{3}{2}}}+\frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\mathrm{erfi}\left(\frac{\sqrt{b}}{x}\right)}{2b^{\frac{3}{2}}}\right)}{4b}$	1

input
```
int(cosh(a+b/x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(b^(1/2)/x)*Pi^(1/2)/exp(a)/b^(1/2)-1/4*exp(a)*Pi^(1/2)/(-b)^(1/2)
*erf((-b)^(1/2)/x)
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = \frac{\sqrt{\pi}\sqrt{-b}(\cosh(a)+\sinh(a))\,\mathrm{erf}\left(\frac{\sqrt{-b}}{x}\right)-\sqrt{\pi}\sqrt{b}(\cosh(a)-\sinh(a))\,\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\,b}$$

input
```
integrate(cosh(a+b/x^2)/x^2,x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)/x) - sqrt(pi)*sqrt
(b)*(cosh(a) - sinh(a))*erf(sqrt(b)/x))/b
```

3.32.6 Sympy [F]

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = \int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

input
```
integrate(cosh(a+b/x**2)/x**2,x)
```

output
```
Integral(cosh(a + b/x**2)/x**2, x)
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = \frac{1}{2}\,b\left(\frac{e^{(-a)}\Gamma\left(\frac{3}{2},\frac{b}{x^2}\right)}{x^3\left(\frac{b}{x^2}\right)^{\frac{3}{2}}}-\frac{e^{a}\Gamma\left(\frac{3}{2},-\frac{b}{x^2}\right)}{x^3\left(-\frac{b}{x^2}\right)^{\frac{3}{2}}}\right)-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{x}$$

input
```
integrate(cosh(a+b/x^2)/x^2,x, algorithm="maxima")
```

output
```
1/2*b*(e^(-a)*gamma(3/2, b/x^2)/(x^3*(b/x^2)^(3/2)) - e^a*gamma(3/2, -b/x^
2)/(x^3*(-b/x^2)^(3/2))) - cosh(a + b/x^2)/x
```

3.32.8 Giac [F]

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx=\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

input
```
integrate(cosh(a+b/x^2)/x^2,x, algorithm="giac")
```

output
```
integrate(cosh(a + b/x^2)/x^2, x)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx=\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

input
```
int(cosh(a + b/x^2)/x^2,x)
```

output
```
int(cosh(a + b/x^2)/x^2, x)
```

3.33 $\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^3}\,dx$

3.33.1 Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = -\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b}$$

output
```
-1/2*sinh(a+b/x^2)/b
```

3.33.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = -\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b}$$

input
```
Integrate[Cosh[a + b/x^2]/x^3,x]
```

output
```
-1/2*Sinh[a + b/x^2]/b
```

3.33.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5844, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^3}\,dx$$

$$\Big\downarrow \text{5844}$$

$$-\frac{1}{2}\int \cosh\left(a+\frac{b}{x^2}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \sin\left(ia+\frac{ib}{x^2}+\frac{\pi}{2}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b}$$

input
```
Int[Cosh[a + b/x^2]/x^3,x]
```

output
```
-1/2*Sinh[a + b/x^2]/b
```

3.33.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

3.33.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b}$	14
default	$-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b}$	14
parallelrisch	$-\frac{\sinh\left(\frac{a\,x^2+b}{x^2}\right)}{2b}$	18
risch	$-\frac{e^{\frac{a\,x^2+b}{x^2}}}{4b}+\frac{e^{-\frac{a\,x^2+b}{x^2}}}{4b}$	37
meijerg	$-\frac{\cosh(a)\sinh\left(\frac{b}{x^2}\right)}{2b}+\frac{\sqrt{\pi}\,\sinh(a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh\left(\frac{b}{x^2}\right)}{\sqrt{\pi}}\right)}{2b}$	40

input
```
int(cosh(a+b/x^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*sinh(a+b/x^2)/b
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = -\frac{\sinh\left(\frac{ax^2+b}{x^2}\right)}{2\,b}$$

input
```
integrate(cosh(a+b/x^2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*sinh((a*x^2 + b)/x^2)/b
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = \begin{cases} -\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b} & \text{for } b \neq 0 \\ -\frac{\cosh(a)}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(a+b/x**2)/x**3,x)
```

output
```
Piecewise((-sinh(a + b/x**2)/(2*b), Ne(b, 0)), (-cosh(a)/(2*x**2), True))
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = -\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2\,b}$$

input
```
integrate(cosh(a+b/x^2)/x^3,x, algorithm="maxima")
```

output
```
-1/2*sinh(a + b/x^2)/b
```

3.33.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = -\frac{e^{\left(\frac{ax^2+b}{x^2}\right)} - e^{\left(-\frac{ax^2+b}{x^2}\right)}}{4\,b}$$

input
```
integrate(cosh(a+b/x^2)/x^3,x, algorithm="giac")
```

output
```
-1/4*(e^((a*x^2 + b)/x^2) - e^(-(a*x^2 + b)/x^2))/b
```

3.33.9 Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^3}\,dx = -\frac{\sinh\left(a + \frac{b}{x^2}\right)}{2\,b}$$

input
```
int(cosh(a + b/x^2)/x^3,x)
```

output
```
-sinh(a + b/x^2)/(2*b)
```

3.34 $\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$

3.34.1 Optimal result

Integrand size = 12, antiderivative size = 75

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = -\frac{e^{-a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}} + \frac{e^{a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}} - \frac{\sinh\left(a+\frac{b}{x^2}\right)}{2bx}$$

output
```
-1/2*sinh(a+b/x^2)/b/x-1/8*erf(b^(1/2)/x)*Pi^(1/2)/b^(3/2)/exp(a)+1/8*exp(
a)*erfi(b^(1/2)/x)*Pi^(1/2)/b^(3/2)
```

3.34.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \frac{\sqrt{\pi}x\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)(-\cosh(a)+\sinh(a)) + \sqrt{\pi}x\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)+\sinh(a)) - 4\sqrt{b}\sinh\left(a+\frac{b}{x^2}\right)}{8b^{3/2}x}$$

input
```
Integrate[Cosh[a + b/x^2]/x^4,x]
```

output
```
(Sqrt[Pi]*x*Erf[Sqrt[b]/x]*(-Cosh[a] + Sinh[a]) + Sqrt[Pi]*x*Erfi[Sqrt[b]/
x]*(Cosh[a] + Sinh[a]) - 4*Sqrt[b]*Sinh[a + b/x^2])/(8*b^(3/2)*x)
```

3.34.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5870, 5848, 5821, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^4}dx$$

$$\Big\downarrow \text{ 5870}$$

$$-\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 5848}$$

$$\frac{\int \sinh\left(a+\frac{b}{x^2}\right)d\frac{1}{x}}{2b}-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2bx}$$

$$\Big\downarrow \text{ 5821}$$

$$\frac{\frac{1}{2}\int e^{a+\frac{b}{x^2}}d\frac{1}{x}-\frac{1}{2}\int e^{-a-\frac{b}{x^2}}d\frac{1}{x}}{2b}-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2bx}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{\frac{\sqrt{\pi}e^a\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}-\frac{1}{2}\int e^{-a-\frac{b}{x^2}}d\frac{1}{x}}{2b}-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2bx}$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{\frac{\sqrt{\pi}e^a\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}e^{-a}\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}}{2b}-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2bx}$$

input
```
Int[Cosh[a + b/x^2]/x^4,x]
```

output
```
(-1/4*(Sqrt[Pi]*Erf[Sqrt[b]/x])/(Sqrt[b]*E^a) + (E^a*Sqrt[Pi]*Erfi[Sqrt[b]
/x])/(4*Sqrt[b]))/(2*b) - Sinh[a + b/x^2]/(2*b*x)
```

3.34.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 5821
```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[1/2   Int[E^(c + d*x^n
), x], x] - Simp[1/2   Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IG
tQ[n, 1]
```

rule 5848
```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1
)/(d*n))   Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[0, n, m + 1]
```

rule 5870
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> -Subst[Int[(a + b*Cosh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

3.34.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

method	result
risch	$\frac{e^{-a} e^{-\frac{b}{x^2}}}{4bx}-\frac{\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)\sqrt{\pi}\, e^{-a}}{8b^{\frac{3}{2}}}-\frac{e^{a} e^{\frac{b}{x^2}}}{4xb}+\frac{e^{a}\sqrt{\pi}\, \operatorname{erf}\left(\frac{\sqrt{-b}}{x}\right)}{8b\sqrt{-b}}$
meijerg	$\frac{\sqrt{\pi}\, \cosh(a)\sqrt{2}\,\sqrt{ib}\left(\frac{\sqrt{2}\,(ib)^{\frac{3}{2}} e^{\frac{b}{x^2}}}{4\sqrt{\pi}\, xb}-\frac{\sqrt{2}\,(ib)^{\frac{3}{2}} e^{-\frac{b}{x^2}}}{4\sqrt{\pi}\, xb}+\frac{(ib)^{\frac{3}{2}}\sqrt{2}\, \operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{\frac{3}{2}}}-\frac{(ib)^{\frac{3}{2}}\sqrt{2}\, \operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{\frac{3}{2}}}\right)}{2b^2}-\frac{i\sqrt{\pi}\, \sinh(a)\sqrt{2}\,\sqrt{ib}\left(\frac{\sqrt{2}\,(ib)^{\frac{5}{2}} e^{-\frac{b}{x^2}}}{4\sqrt{\pi}\, x\, b^2}\right.}{}$

input
```
int(cosh(a+b/x^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/4/exp(a)/b/x*exp(-b/x^2)-1/8*erf(b^(1/2)/x)*Pi^(1/2)/b^(3/2)/exp(a)-1/4*
exp(a)*exp(b/x^2)/x/b+1/8*exp(a)/b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

3.34.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(55) = 110.

Time = 0.25 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.33

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx =$$
$$-\frac{2\,b\cosh\left(\frac{ax^2+b}{x^2}\right)^2+\sqrt{\pi}\left(x\cosh\left(a\right)\cosh\left(\frac{ax^2+b}{x^2}\right)+x\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh\left(a\right)+\left(x\cosh\left(a\right)+x\sinh\left(a\right)\right)\mathrm{si}}{}$$

input
```
integrate(cosh(a+b/x^2)/x^4,x, algorithm="fricas")
```

output
```
-1/8*(2*b*cosh((a*x^2 + b)/x^2)^2 + sqrt(pi)*(x*cosh(a)*cosh((a*x^2 + b)/x
^2) + x*cosh((a*x^2 + b)/x^2)*sinh(a) + (x*cosh(a) + x*sinh(a))*sinh((a*x^
2 + b)/x^2))*sqrt(-b)*erf(sqrt(-b)/x) + sqrt(pi)*(x*cosh(a)*cosh((a*x^2 +
b)/x^2) - x*cosh((a*x^2 + b)/x^2)*sinh(a) + (x*cosh(a) - x*sinh(a))*sinh((
a*x^2 + b)/x^2))*sqrt(b)*erf(sqrt(b)/x) + 4*b*cosh((a*x^2 + b)/x^2)*sinh((
a*x^2 + b)/x^2) + 2*b*sinh((a*x^2 + b)/x^2)^2 - 2*b)/(b^2*x*cosh((a*x^2 +
b)/x^2) + b^2*x*sinh((a*x^2 + b)/x^2))
```

3.34.6 Sympy [F]

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

input
```
integrate(cosh(a+b/x**2)/x**4,x)
```

output
```
Integral(cosh(a + b/x**2)/x**4, x)
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \frac{1}{6}\,b\left(\frac{e^{(-a)}\Gamma\left(\frac{5}{2},\frac{b}{x^2}\right)}{x^5\left(\frac{b}{x^2}\right)^{\frac{5}{2}}} - \frac{e^{a}\Gamma\left(\frac{5}{2},-\frac{b}{x^2}\right)}{x^5\left(-\frac{b}{x^2}\right)^{\frac{5}{2}}}\right) - \frac{\cosh\left(a+\frac{b}{x^2}\right)}{3\,x^3}$$

input
```
integrate(cosh(a+b/x^2)/x^4,x, algorithm="maxima")
```

output
```
1/6*b*(e^(-a)*gamma(5/2, b/x^2)/(x^5*(b/x^2)^(5/2)) - e^a*gamma(5/2, -b/x^
2)/(x^5*(-b/x^2)^(5/2))) - 1/3*cosh(a + b/x^2)/x^3
```

3.34.8 Giac [F]

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

input
```
integrate(cosh(a+b/x^2)/x^4,x, algorithm="giac")
```

output
```
integrate(cosh(a + b/x^2)/x^4, x)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

input
```
int(cosh(a + b/x^2)/x^4,x)
```

output
```
int(cosh(a + b/x^2)/x^4, x)
```

3.35 $\int \cosh(a + bx^n)\,dx$

3.35.1 Optimal result

Integrand size = 8, antiderivative size = 67

$$\int \cosh(a + bx^n)\,dx = -\frac{e^a x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n}, -bx^n\right)}{2n} - \frac{e^{-a}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n}, bx^n\right)}{2n}$$

output
```
-1/2*exp(a)*x*GAMMA(1/n,-b*x^n)/n/((-b*x^n)^(1/n))-1/2*x*GAMMA(1/n,b*x^n)/
exp(a)/n/((b*x^n)^(1/n))
```

3.35.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \cosh(a + bx^n)\,dx = -\frac{e^a x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n}, -bx^n\right) + e^{-a}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n}, bx^n\right)}{2n}$$

input
```
Integrate[Cosh[a + b*x^n],x]
```

output
```
-1/2*((E^a*x*Gamma[n^(-1), -(b*x^n)])/(-(b*x^n))^n^(-1) + (x*Gamma[n^(-1),
 b*x^n])/(E^a*(b*x^n)^n^(-1)))/n
```

3.35.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5830, 2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh \left(a+bx^n\right) \, dx$$

$$\Big\downarrow \text{5830}$$

$$\frac{1}{2}\int e^{-bx^n-a}dx+\frac{1}{2}\int e^{bx^n+a}dx$$

$$\Big\downarrow \text{2637}$$

$$-\frac{e^a x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-bx^n\right)}{2n}-\frac{e^{-a}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},bx^n\right)}{2n}$$

input
```
Int[Cosh[a + b*x^n],x]
```

output
```
-1/2*(E^a*x*Gamma[n^(-1), -(b*x^n)])/(n*(-(b*x^n))^n^(-1)) - (x*Gamma[n^(-
1), b*x^n])/(2*E^a*n*(b*x^n)^n^(-1))
```

3.35.3.1 Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

rule 5830
```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[1/2   Int[E^(c + d*x^n
), x], x] + Simp[1/2   Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d, n}, x]
```

3.35.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

method	result	siz
meijerg	$x \,\mathrm{hypergeom}\left(\left[\frac{1}{2n}\right], \left[\frac{1}{2}, 1+\frac{1}{2n}\right], \frac{x^{2n} b^{2}}{4}\right) \cosh\left(a\right)+\frac{x^{n+1} b \,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{1}{2n}\right], \left[\frac{3}{2}, \frac{3}{2}+\frac{1}{2n}\right], \frac{x^{2n} b^{2}}{4}\right) \sinh\left(a\right)}{n+1}$	74

input
```
int(cosh(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/2/n],[1/2,1+1/2/n],1/4*x^(2*n)*b^2)*cosh(a)+1/(n+1)*x^(n+1)
*b*hypergeom([1/2+1/2/n],[3/2,3/2+1/2/n],1/4*x^(2*n)*b^2)*sinh(a)
```

3.35.5 Fricas [F]

$$\int \cosh\left(a+bx^{n}\right)\,dx = \int \cosh\left(bx^{n}+a\right)\,dx$$

input
```
integrate(cosh(a+b*x^n),x, algorithm="fricas")
```

output
```
integral(cosh(b*x^n + a), x)
```

3.35.6 Sympy [F]

$$\int \cosh\left(a+bx^{n}\right)\,dx = \int \cosh\left(a+bx^{n}\right)dx$$

input
```
integrate(cosh(a+b*x**n),x)
```

output
```
Integral(cosh(a + b*x**n), x)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \cosh \left(a+bx^{n}\right) \, dx=-\frac{xe^{(-a)}\Gamma\left(\frac{1}{n},bx^{n}\right)}{2\left(bx^{n}\right)^{\left(\frac{1}{n}\right)}n}-\frac{xe^{a}\Gamma\left(\frac{1}{n},-bx^{n}\right)}{2\left(-bx^{n}\right)^{\left(\frac{1}{n}\right)}n}$$

input
```
integrate(cosh(a+b*x^n),x, algorithm="maxima")
```

output
```
-1/2*x*e^(-a)*gamma(1/n, b*x^n)/((b*x^n)^(1/n)*n) - 1/2*x*e^a*gamma(1/n, -
b*x^n)/((-b*x^n)^(1/n)*n)
```

3.35.8 Giac [F]

$$\int \cosh \left(a+bx^{n}\right) \, dx=\int \cosh \left(bx^{n}+a\right) \, dx$$

input
```
integrate(cosh(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(cosh(b*x^n + a), x)
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \cosh \left(a+bx^{n}\right) \, dx=\int \cosh(a+b\,x^{n}) \, dx$$

input
```
int(cosh(a + b*x^n),x)
```

output
```
int(cosh(a + b*x^n), x)
```

3.36 $\int \frac{\cosh(a+bx^n)}{x}\,dx$

3.36.1 Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{\cosh(a+bx^n)}{x}\,dx = \frac{\cosh(a)\mathrm{Chi}(bx^n)}{n} + \frac{\sinh(a)\mathrm{Shi}(bx^n)}{n}$$

output
```
Chi(b*x^n)*cosh(a)/n+Shi(b*x^n)*sinh(a)/n
```

3.36.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cosh(a+bx^n)}{x}\,dx = \frac{\cosh(a)\mathrm{Chi}(bx^n) + \sinh(a)\mathrm{Shi}(bx^n)}{n}$$

input
```
Integrate[Cosh[a + b*x^n]/x,x]
```

output
```
(Cosh[a]*CoshIntegral[b*x^n] + Sinh[a]*SinhIntegral[b*x^n])/n
```

3.36.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5842, 5839, 5840}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx^n)}{x} dx$$

$$\Big\downarrow \text{ 5842}$$

$$\sinh(a)\int \frac{\sinh(bx^n)}{x}dx+\cosh(a)\int \frac{\cosh(bx^n)}{x}dx$$

$$\Big\downarrow \text{ 5839}$$

$$\cosh(a)\int \frac{\cosh(bx^n)}{x}dx+\frac{\sinh(a)\text{Shi}(bx^n)}{n}$$

$$\Big\downarrow \text{ 5840}$$

$$\frac{\cosh(a)\text{Chi}(bx^n)}{n}+\frac{\sinh(a)\text{Shi}(bx^n)}{n}$$

input
```
Int[Cosh[a + b*x^n]/x,x]
```

output
```
(Cosh[a]*CoshIntegral[b*x^n])/n + (Sinh[a]*SinhIntegral[b*x^n])/n
```

3.36.3.1 Defintions of rubi rules used

rule 5839
```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

rule 5840
```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

rule 5842
```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[Cosh[c]   Int[Cosh
[d*x^n]/x, x], x] + Simp[Sinh[c]   Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d
, n}, x]
```

3.36.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

method	result	size
risch	$-\frac{{\mathrm e}^{-a} \operatorname{Ei}_{1}\left(b \,x^{n}\right)}{2 n}-\frac{{\mathrm e}^{a} \operatorname{Ei}_{1}\left(-b \,x^{n}\right)}{2 n}$	33
meijerg	$\frac{\sqrt{\pi}\, \left(\frac{2 \gamma +2 n \ln \left(x \right)+2 \ln \left(i b \right)}{\sqrt{\pi}}+\frac{2 \,\operatorname{Chi}\left(b \,x^{n}\right)-2 \ln \left(b \,x^{n}\right)-2 \gamma}{\sqrt{\pi}}\right) \cosh \left(a \right)}{2 n}+\frac{\operatorname{Shi}\left(b \,x^{n}\right) \sinh \left(a \right)}{n}$	68

input
```
int(cosh(a+b*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2/n*exp(-a)*Ei(1,b*x^n)-1/2/n*exp(a)*Ei(1,-b*x^n)
```

3.36.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \frac{\cosh\left(a+bx^n\right)}{x}\,dx = \frac{(\cosh\left(a\right)+\sinh\left(a\right))\mathrm{Ei}(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right))+(\cosh\left(a\right)-\sinh\left(a\right))\mathrm{Ei}(-b\cosh\left(n\log\left(x\right)\right)}{2\,n}$$

input
```
integrate(cosh(a+b*x^n)/x,x, algorithm="fricas")
```

output
```
1/2*((cosh(a) + sinh(a))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) + (cosh(a
) - sinh(a))*Ei(-b*cosh(n*log(x)) - b*sinh(n*log(x))))/n
```

3.36.6 Sympy [F]

$$\int \frac{\cosh\left(a+bx^n\right)}{x}\,dx = \int \frac{\cosh\left(a+bx^n\right)}{x}\,dx$$

input
```
integrate(cosh(a+b*x**n)/x,x)
```

output
```
Integral(cosh(a + b*x**n)/x, x)
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{\cosh(a+bx^n)}{x}\,dx = \frac{\mathrm{Ei}(-bx^n)\,e^{(-a)}}{2\,n} + \frac{\mathrm{Ei}(bx^n)\,e^a}{2\,n}$$

input
```
integrate(cosh(a+b*x^n)/x,x, algorithm="maxima")
```

output
```
1/2*Ei(-b*x^n)*e^(-a)/n + 1/2*Ei(b*x^n)*e^a/n
```

3.36.8 Giac [F]

$$\int \frac{\cosh(a+bx^n)}{x}\,dx = \int \frac{\cosh(bx^n+a)}{x}\,dx$$

input
```
integrate(cosh(a+b*x^n)/x,x, algorithm="giac")
```

output
```
integrate(cosh(b*x^n + a)/x, x)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx^n)}{x}\,dx = \int \frac{\cosh(a+b\,x^n)}{x}\,dx$$

input
```
int(cosh(a + b*x^n)/x,x)
```

output
```
int(cosh(a + b*x^n)/x, x)
```

3.37 $\int \cosh^2(a+bx^n)\,dx$

3.37.1 Optimal result

Integrand size = 10, antiderivative size = 89

$$\int \cosh^2(a+bx^n)\,dx = \frac{x}{2} - \frac{2^{-2-\frac{1}{n}}e^{2a}x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-2bx^n\right)}{n} - \frac{2^{-2-\frac{1}{n}}e^{-2a}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},2bx^n\right)}{n}$$

output
```
1/2*x-2^(-2-1/n)*exp(2*a)*x*GAMMA(1/n,-2*b*x^n)/n/((-b*x^n)^(1/n))-2^(-2-1
/n)*x*GAMMA(1/n,2*b*x^n)/exp(2*a)/n/((b*x^n)^(1/n))
```

3.37.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \cosh^2(a+bx^n)\,dx = -\frac{x\left(-2n+2^{-1/n}e^{2a}(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-2bx^n\right)+2^{-1/n}e^{-2a}(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},2bx^n\right)\right)}{4n}$$

input
```
Integrate[Cosh[a + b*x^n]^2,x]
```

output
```
-1/4*(x*(-2*n + (E^(2*a)*Gamma[n^(-1), -2*b*x^n])/(2^n^(-1)*(-(b*x^n))^n^(
-1)) + Gamma[n^(-1), 2*b*x^n]/(2^n^(-1)*E^(2*a)*(b*x^n)^n^(-1))))/n
```

3.37.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5832, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2\left(a+bx^n\right)\,dx$$

$$\Big\downarrow \text{ 5832}$$

$$\int \left(\frac{1}{2}\cosh\left(2a+2bx^n\right)+\frac{1}{2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{e^{2a}2^{-\frac{1}{n}-2}x\left(-bx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-2bx^n\right)}{n}-\frac{e^{-2a}2^{-\frac{1}{n}-2}x\left(bx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},2bx^n\right)}{n}+\frac{x}{2}$$

input
```
Int[Cosh[a + b*x^n]^2,x]
```

output
```
x/2 - (2^(-2 - n^(-1))*E^(2*a)*x*Gamma[n^(-1), -2*b*x^n])/(n*(-(b*x^n))^n^
(-1)) - (2^(-2 - n^(-1))*x*Gamma[n^(-1), 2*b*x^n])/(E^(2*a)*n*(b*x^n)^n^(-
1))
```

3.37.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5832
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[p, 0]
```

3.37.4 Maple [F]

$$\int \cosh\left(a+b\,x^{n}\right)^{2}dx$$

input
```
int(cosh(a+b*x^n)^2,x)
```

output
```
int(cosh(a+b*x^n)^2,x)
```

3.37.5 Fricas [F]

$$\int \cosh^2\left(a+bx^n\right)\,dx=\int \cosh\left(bx^n+a\right)^2\,dx$$

input
```
integrate(cosh(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral(cosh(b*x^n + a)^2, x)
```

3.37.6 Sympy [F]

$$\int \cosh^2\left(a+bx^n\right)\,dx=\int \cosh^2\left(a+bx^n\right)dx$$

input
```
integrate(cosh(a+b*x**n)**2,x)
```

output
```
Integral(cosh(a + b*x**n)**2, x)
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \cosh^2\left(a+bx^n\right)\,dx=\frac{1}{2}\,x-\frac{xe^{(-2\,a)}\Gamma\left(\frac{1}{n},2\,bx^n\right)}{4\,\left(2\,bx^n\right)^{\left(\frac{1}{n}\right)}n}-\frac{xe^{(2\,a)}\Gamma\left(\frac{1}{n},-2\,bx^n\right)}{4\,\left(-2\,bx^n\right)^{\left(\frac{1}{n}\right)}n}$$

input
```
integrate(cosh(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
1/2*x - 1/4*x*e^(-2*a)*gamma(1/n, 2*b*x^n)/((2*b*x^n)^(1/n)*n) - 1/4*x*e^(
2*a)*gamma(1/n, -2*b*x^n)/((-2*b*x^n)^(1/n)*n)
```

3.37.8 Giac [F]

$$\int \cosh^2\left(a+bx^n\right)\,dx=\int \cosh\left(bx^n+a\right)^2\,dx$$

input
```
integrate(cosh(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(cosh(b*x^n + a)^2, x)
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \cosh^2\left(a+bx^n\right)\,dx=\int \cosh(a+b\,x^n)^2\,dx$$

input
```
int(cosh(a + b*x^n)^2,x)
```

output
```
int(cosh(a + b*x^n)^2, x)
```

3.38 $\int \frac{\cosh^2(a+bx^n)}{x} \, dx$

3.38.1 Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \frac{\cosh^2(a+bx^n)}{x} \, dx = \frac{\cosh(2a)\mathrm{Chi}(2bx^n)}{2n} + \frac{\log(x)}{2} + \frac{\sinh(2a)\mathrm{Shi}(2bx^n)}{2n}$$

output
```
1/2*Chi(2*b*x^n)*cosh(2*a)/n+1/2*ln(x)+1/2*Shi(2*b*x^n)*sinh(2*a)/n
```

3.38.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\cosh^2(a+bx^n)}{x} \, dx = \frac{\cosh(2a)\mathrm{Chi}(2bx^n) + n\log(x) + \sinh(2a)\mathrm{Shi}(2bx^n)}{2n}$$

input
```
Integrate[Cosh[a + b*x^n]^2/x,x]
```

output
```
(Cosh[2*a]*CoshIntegral[2*b*x^n] + n*Log[x] + Sinh[2*a]*SinhIntegral[2*b*x
^n])/(2*n)
```

3.38.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5886, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(a+bx^n)}{x} \, dx$$

$$\Big\downarrow \text{5886}$$

$$\int \left(\frac{\cosh(2a+2bx^n)}{2x}+\frac{1}{2x}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cosh(2a)\text{Chi}(2bx^n)}{2n}+\frac{\sinh(2a)\text{Shi}(2bx^n)}{2n}+\frac{\log(x)}{2}$$

input
```
Int[Cosh[a + b*x^n]^2/x,x]
```

output
```
(Cosh[2*a]*CoshIntegral[2*b*x^n])/(2*n) + Log[x]/2 + (Sinh[2*a]*SinhIntegr
al[2*b*x^n])/(2*n)
```

3.38.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5886
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

3.38.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
risch	$\frac{\ln(x)}{2}-\frac{\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2b\,x^n)}{4n}-\frac{\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2b\,x^n)}{4n}$	40

input
```
int(cosh(a+b*x^n)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x)-1/4/n*exp(-2*a)*Ei(1,2*b*x^n)-1/4/n*exp(2*a)*Ei(1,-2*b*x^n)
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \frac{\cosh^2(a+bx^n)}{x}\,dx = \frac{(\cosh(2a)+\sinh(2a))\mathrm{Ei}(2b\cosh(n\log(x))+2b\sinh(n\log(x)))+(\cosh(2a)-\sinh(2a))\mathrm{Ei}(-2b\cosh}{4n}$$

input
```
integrate(cosh(a+b*x^n)^2/x,x, algorithm="fricas")
```

output
```
1/4*((cosh(2*a) + sinh(2*a))*Ei(2*b*cosh(n*log(x)) + 2*b*sinh(n*log(x))) +
 (cosh(2*a) - sinh(2*a))*Ei(-2*b*cosh(n*log(x)) - 2*b*sinh(n*log(x))) + 2*
n*log(x))/n
```

3.38.6 Sympy [F]

$$\int \frac{\cosh^2(a+bx^n)}{x}\,dx = \int \frac{\cosh^2(a+bx^n)}{x}\,dx$$

input
```
integrate(cosh(a+b*x**n)**2/x,x)
```

output
```
Integral(cosh(a + b*x**n)**2/x, x)
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\cosh^2(a+bx^n)}{x}\,dx = \frac{\mathrm{Ei}(2\,bx^n)\,e^{(2\,a)}}{4\,n} + \frac{\mathrm{Ei}(-2\,bx^n)\,e^{(-2\,a)}}{4\,n} + \frac{1}{2}\log(x)$$

input
```
integrate(cosh(a+b*x^n)^2/x,x, algorithm="maxima")
```

output
```
1/4*Ei(2*b*x^n)*e^(2*a)/n + 1/4*Ei(-2*b*x^n)*e^(-2*a)/n + 1/2*log(x)
```

3.38.8 Giac [F]

$$\int \frac{\cosh^2(a+bx^n)}{x}\,dx = \int \frac{\cosh(bx^n+a)^2}{x}\,dx$$

input
```
integrate(cosh(a+b*x^n)^2/x,x, algorithm="giac")
```

output
```
integrate(cosh(b*x^n + a)^2/x, x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx^n)}{x}\,dx = \int \frac{\cosh(a+b\,x^n)^2}{x}\,dx$$

input
```
int(cosh(a + b*x^n)^2/x,x)
```

output
```
int(cosh(a + b*x^n)^2/x, x)
```

3.39 $\int \cosh^3(a+bx^n)\,dx$

3.39.1 Optimal result

Integrand size = 10, antiderivative size = 150

$$\int \cosh^3(a+bx^n)\,dx = -\frac{3^{-1/n}e^{3a}x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-3bx^n\right)}{8n}-\frac{3e^ax(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},bx^n\right)}{8n}-\frac{3^{-1/n}e^{-3a}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},3bx^n\right)}{8n}$$

output
```
-1/8*exp(3*a)*x*GAMMA(1/n,-3*b*x^n)/(3^(1/n))/n/((-b*x^n)^(1/n))-3/8*exp(a
)*x*GAMMA(1/n,-b*x^n)/n/((-b*x^n)^(1/n))-3/8*x*GAMMA(1/n,b*x^n)/exp(a)/n/(
(b*x^n)^(1/n))-1/8*x*GAMMA(1/n,3*b*x^n)/(3^(1/n))/exp(3*a)/n/((b*x^n)^(1/n
))
```

3.39.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\int \cosh^3(a+bx^n)\,dx = -\frac{3^{-1/n}e^{-3a}x(-b^2x^{2n})^{-1/n}\left(e^{6a}(bx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-3bx^n\right)+3^{1+\frac{1}{n}}e^{4a}(bx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-bx^n\right)+(-bx^n)^{\frac{1}{n}}\left(3^{1+\frac{1}{n}}e^{2a}\Gamma(\right.\right.}{8n}$$

input
```
Integrate[Cosh[a + b*x^n]^3,x]
```

output
```
-1/8*(x*(E^(6*a)*(b*x^n)^n^(-1)*Gamma[n^(-1), -3*b*x^n] + 3^(1 + n^(-1))*E
^(4*a)*(b*x^n)^n^(-1)*Gamma[n^(-1), -(b*x^n)] + (-(b*x^n))^n^(-1)*(3^(1 +
n^(-1))*E^(2*a)*Gamma[n^(-1), b*x^n] + Gamma[n^(-1), 3*b*x^n])))/(3^n^(-1)
*E^(3*a)*n*(-(b^2*x^(2*n)))^n^(-1))
```

3.39.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5832, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3\left(a+bx^n\right)\,dx$$

$$\Big\downarrow \; 5832$$

$$\int \left(\frac{3}{4}\cosh\left(a+bx^n\right)+\frac{1}{4}\cosh\left(3a+3bx^n\right)\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{e^{3a}3^{-1/n}x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-3bx^n\right)}{8n}-\frac{3e^{a}x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},bx^n\right)}{8n}-\frac{e^{-3a}3^{-1/n}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},3bx^n\right)}{8n}$$

input
```
Int[Cosh[a + b*x^n]^3,x]
```

output
```
-1/8*(E^(3*a)*x*Gamma[n^(-1), -3*b*x^n])/(3^n^(-1)*n*(-(b*x^n))^n^(-1)) -
(3*E^a*x*Gamma[n^(-1), -(b*x^n)])/(8*n*(-(b*x^n))^n^(-1)) - (3*x*Gamma[n^(
-1), b*x^n])/(8*E^a*n*(b*x^n)^n^(-1)) - (x*Gamma[n^(-1), 3*b*x^n])/(8*3^n^
(-1)*E^(3*a)*n*(b*x^n)^n^(-1))
```

3.39.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5832
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[p, 0]
```

3.39.4 Maple [F]

$$\int \cosh \left(a+b\,x^{n}\right)^{3}dx$$

input
```
int(cosh(a+b*x^n)^3,x)
```

output
```
int(cosh(a+b*x^n)^3,x)
```

3.39.5 Fricas [F]

$$\int \cosh^3\left(a+bx^n\right)\,dx=\int \cosh\left(bx^n+a\right)^3\,dx$$

input
```
integrate(cosh(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral(cosh(b*x^n + a)^3, x)
```

3.39.6 Sympy [F]

$$\int \cosh^3\left(a+bx^n\right)\,dx=\int \cosh^3\left(a+bx^n\right)dx$$

input
```
integrate(cosh(a+b*x**n)**3,x)
```

output
```
Integral(cosh(a + b*x**n)**3, x)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

$$\int \cosh^3(a+bx^n)\,dx = -\frac{xe^{(-3\,a)}\Gamma\left(\frac{1}{n},3\,bx^n\right)}{8\,(3\,bx^n)^{\left(\frac{1}{n}\right)}n} - \frac{3\,xe^{(-a)}\Gamma\left(\frac{1}{n},bx^n\right)}{8\,(bx^n)^{\left(\frac{1}{n}\right)}n} - \frac{3\,xe^{a}\Gamma\left(\frac{1}{n},-bx^n\right)}{8\,(-bx^n)^{\left(\frac{1}{n}\right)}n} - \frac{xe^{(3\,a)}\Gamma\left(\frac{1}{n},-3\,bx^n\right)}{8\,(-3\,bx^n)^{\left(\frac{1}{n}\right)}n}$$

input
```
integrate(cosh(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
-1/8*x*e^(-3*a)*gamma(1/n, 3*b*x^n)/((3*b*x^n)^(1/n)*n) - 3/8*x*e^(-a)*gam
ma(1/n, b*x^n)/((b*x^n)^(1/n)*n) - 3/8*x*e^a*gamma(1/n, -b*x^n)/((-b*x^n)^
(1/n)*n) - 1/8*x*e^(3*a)*gamma(1/n, -3*b*x^n)/((-3*b*x^n)^(1/n)*n)
```

3.39.8 Giac [F]

$$\int \cosh^3(a+bx^n)\,dx = \int \cosh(bx^n+a)^3\,dx$$

input
```
integrate(cosh(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(cosh(b*x^n + a)^3, x)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \cosh^3(a+bx^n)\,dx = \int \cosh(a+b\,x^n)^3\,dx$$

input
```
int(cosh(a + b*x^n)^3,x)
```

output
```
int(cosh(a + b*x^n)^3, x)
```

3.40 $\int \frac{\cosh^3(a+bx^n)}{x} dx$

3.40.1 Optimal result

Integrand size = 14, antiderivative size = 67

$$\int \frac{\cosh^3(a+bx^n)}{x} dx = \frac{3\cosh(a)\text{Chi}(bx^n)}{4n} + \frac{\cosh(3a)\text{Chi}(3bx^n)}{4n} + \frac{3\sinh(a)\text{Shi}(bx^n)}{4n} + \frac{\sinh(3a)\text{Shi}(3bx^n)}{4n}$$

output
```
3/4*Chi(b*x^n)*cosh(a)/n+1/4*Chi(3*b*x^n)*cosh(3*a)/n+3/4*Shi(b*x^n)*sinh(
a)/n+1/4*Shi(3*b*x^n)*sinh(3*a)/n
```

3.40.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \frac{\cosh^3(a+bx^n)}{x} dx = \frac{3\cosh(a)\text{Chi}(bx^n) + \cosh(3a)\text{Chi}(3bx^n) + 3\sinh(a)\text{Shi}(bx^n) + \sinh(3a)\text{Shi}(3bx^n)}{4n}$$

input
```
Integrate[Cosh[a + b*x^n]^3/x,x]
```

output
```
(3*Cosh[a]*CoshIntegral[b*x^n] + Cosh[3*a]*CoshIntegral[3*b*x^n] + 3*Sinh[
a]*SinhIntegral[b*x^n] + Sinh[3*a]*SinhIntegral[3*b*x^n])/(4*n)
```

3.40.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5886, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(a+bx^n)}{x} \, dx$$

$$\Big\downarrow \text{5886}$$

$$\int \left(\frac{3\cosh(a+bx^n)}{4x}+\frac{\cosh(3a+3bx^n)}{4x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\cosh(a)\text{Chi}(bx^n)}{4n}+\frac{\cosh(3a)\text{Chi}(3bx^n)}{4n}+\frac{3\sinh(a)\text{Shi}(bx^n)}{4n}+\frac{\sinh(3a)\text{Shi}(3bx^n)}{4n}$$

input
```
Int[Cosh[a + b*x^n]^3/x,x]
```

output
```
(3*Cosh[a]*CoshIntegral[b*x^n])/(4*n) + (Cosh[3*a]*CoshIntegral[3*b*x^n])/
(4*n) + (3*Sinh[a]*SinhIntegral[b*x^n])/(4*n) + (Sinh[3*a]*SinhIntegral[3*
b*x^n])/(4*n)
```

3.40.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5886
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

3.40.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{e^{-3a}\operatorname{Ei}_1(3b\,x^n)}{8n}-\frac{3\,e^{-a}\operatorname{Ei}_1(b\,x^n)}{8n}-\frac{e^{3a}\operatorname{Ei}_1(-3b\,x^n)}{8n}-\frac{3\,e^{a}\operatorname{Ei}_1(-b\,x^n)}{8n}$	67

input
```
int(cosh(a+b*x^n)^3/x,x,method=_RETURNVERBOSE)
```

output
```
-1/8/n*exp(-3*a)*Ei(1,3*b*x^n)-3/8/n*exp(-a)*Ei(1,b*x^n)-1/8/n*exp(3*a)*Ei
(1,-3*b*x^n)-3/8/n*exp(a)*Ei(1,-b*x^n)
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70

$$\int \frac{\cosh^3(a+bx^n)}{x}\,dx$$
$$= \frac{(\cosh(3a)+\sinh(3a))\mathrm{Ei}(3b\cosh(n\log(x))+3b\sinh(n\log(x)))+3(\cosh(a)+\sinh(a))\mathrm{Ei}(b\cosh(n\log$$

input
```
integrate(cosh(a+b*x^n)^3/x,x, algorithm="fricas")
```

output
```
1/8*((cosh(3*a) + sinh(3*a))*Ei(3*b*cosh(n*log(x)) + 3*b*sinh(n*log(x))) +
 3*(cosh(a) + sinh(a))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) + 3*(cosh(a
) - sinh(a))*Ei(-b*cosh(n*log(x)) - b*sinh(n*log(x))) + (cosh(3*a) - sinh(
3*a))*Ei(-3*b*cosh(n*log(x)) - 3*b*sinh(n*log(x))))/n
```

3.40.6 Sympy [F]

$$\int \frac{\cosh^3(a+bx^n)}{x}\,dx = \int \frac{\cosh^3(a+bx^n)}{x}\,dx$$

input
```
integrate(cosh(a+b*x**n)**3/x,x)
```

output
```
Integral(cosh(a + b*x**n)**3/x, x)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \frac{\cosh^3(a+bx^n)}{x}\,dx = \frac{\mathrm{Ei}(3\,bx^n)\,e^{(3\,a)}}{8\,n} + \frac{3\,\mathrm{Ei}(-bx^n)\,e^{(-a)}}{8\,n} + \frac{\mathrm{Ei}(-3\,bx^n)\,e^{(-3\,a)}}{8\,n} + \frac{3\,\mathrm{Ei}(bx^n)\,e^{a}}{8\,n}$$

input
```
integrate(cosh(a+b*x^n)^3/x,x, algorithm="maxima")
```

output
```
1/8*Ei(3*b*x^n)*e^(3*a)/n + 3/8*Ei(-b*x^n)*e^(-a)/n + 1/8*Ei(-3*b*x^n)*e^(
-3*a)/n + 3/8*Ei(b*x^n)*e^a/n
```

3.40.8 Giac [F]

$$\int \frac{\cosh^3(a+bx^n)}{x}\,dx = \int \frac{\cosh(bx^n+a)^3}{x}\,dx$$

input
```
integrate(cosh(a+b*x^n)^3/x,x, algorithm="giac")
```

output
```
integrate(cosh(b*x^n + a)^3/x, x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx^n)}{x}\,dx = \int \frac{\cosh(a+b\,x^n)^3}{x}\,dx$$

input
```
int(cosh(a + b*x^n)^3/x,x)
```

output
```
int(cosh(a + b*x^n)^3/x, x)
```

3.41 $\int (ex)^m (b\cosh(c+dx^n))^p \, dx$

3.41.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (ex)^m (b\cosh(c+dx^n))^p \, dx = \text{Int}((ex)^m (b\cosh(c+dx^n))^p, x)$$

output
```
Unintegrable((e*x)^m*(b*cosh(c+d*x^n))^p,x)
```

3.41.2 Mathematica [N/A]

Not integrable

Time = 4.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\cosh(c+dx^n))^p \, dx = \int (ex)^m (b\cosh(c+dx^n))^p \, dx$$

input
```
Integrate[(e*x)^m*(b*Cosh[c + d*x^n])^p,x]
```

output
```
Integrate[(e*x)^m*(b*Cosh[c + d*x^n])^p, x]
```

3.41.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5890}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (b\cosh(c+dx^n))^p\,dx$$

$$\Big\downarrow \text{ 5890}$$

$$\int (ex)^m (b\cosh(c+dx^n))^p dx$$

input
```
Int[(e*x)^m*(b*Cosh[c + d*x^n])^p,x]
```

output
```
$Aborted
```

3.41.3.1 Defintions of rubi rules used

rule 5890
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_))^(m_.),
 x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Cosh[c + d*u^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && LinearQ[u, x]
```

3.41.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (ex)^m (b\cosh(c+d\,x^n))^p dx$$

input
```
int((e*x)^m*(b*cosh(c+d*x^n))^p,x)
```

output
```
int((e*x)^m*(b*cosh(c+d*x^n))^p,x)
```

3.41.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\cosh(c+dx^n))^p \, dx = \int (ex)^m (b\cosh(dx^n+c))^p \, dx$$

input
```
integrate((e*x)^m*(b*cosh(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*(b*cosh(d*x^n + c))^p, x)
```

3.41.6 Sympy [N/A]

Not integrable

Time = 8.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (ex)^m (b\cosh(c+dx^n))^p \, dx = \int (b\cosh(c+dx^n))^p (ex)^m \, dx$$

input
```
integrate((e*x)**m*(b*cosh(c+d*x**n))**p,x)
```

output
```
Integral((b*cosh(c + d*x**n))**p*(e*x)**m, x)
```

3.41.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\cosh(c+dx^n))^p \, dx = \int (ex)^m (b\cosh(dx^n+c))^p \, dx$$

input
```
integrate((e*x)^m*(b*cosh(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*(b*cosh(d*x^n + c))^p, x)
```

3.41.8 Giac [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\cosh (c+dx^n))^p \, dx = \int (ex)^m (b\cosh (dx^n+c))^p \, dx$$

input
```
integrate((e*x)^m*(b*cosh(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*(b*cosh(d*x^n + c))^p, x)
```

3.41.9 Mupad [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\cosh (c+dx^n))^p \, dx = \int (e\,x)^m\,(b\cosh(c+d\,x^n))^p \,dx$$

input
```
int((e*x)^m*(b*cosh(c + d*x^n))^p,x)
```

output
```
int((e*x)^m*(b*cosh(c + d*x^n))^p, x)
```

3.42 $\int (ex)^m (a+b\cosh(c+dx^n))^p \, dx$

3.42.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (ex)^m (a+b\cosh(c+dx^n))^p \, dx = \mathrm{Int}((ex)^m (a+b\cosh(c+dx^n))^p, x)$$

output
```
Unintegrable((e*x)^m*(a+b*cosh(c+d*x^n))^p,x)
```

3.42.2 Mathematica [N/A]

Not integrable

Time = 6.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\cosh(c+dx^n))^p \, dx = \int (ex)^m (a+b\cosh(c+dx^n))^p \, dx$$

input
```
Integrate[(e*x)^m*(a + b*Cosh[c + d*x^n])^p,x]
```

output
```
Integrate[(e*x)^m*(a + b*Cosh[c + d*x^n])^p, x]
```

3.42.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5890}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a + b\cosh(c + dx^n))^p\, dx$$
$$\Big\downarrow \text{5890}$$
$$\int (ex)^m (a + b\cosh(c + dx^n))^p dx$$

input
```
Int[(e*x)^m*(a + b*Cosh[c + d*x^n])^p,x]
```

output
```
$Aborted
```

3.42.3.1 Defintions of rubi rules used

rule 5890
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_))^(m_.),
 x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Cosh[c + d*u^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && LinearQ[u, x]
```

3.42.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (ex)^m (a + b\cosh(c + d\,x^n))^p dx$$

input
```
int((e*x)^m*(a+b*cosh(c+d*x^n))^p,x)
```

output
```
int((e*x)^m*(a+b*cosh(c+d*x^n))^p,x)
```

3.42.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\cosh (c + dx^n))^p \, dx = \int (ex)^m (b\cosh (dx^n + c) + a)^p \, dx$$

input
```
integrate((e*x)^m*(a+b*cosh(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*(b*cosh(d*x^n + c) + a)^p, x)
```

3.42.6 Sympy [N/A]

Not integrable

Time = 25.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (ex)^m (a + b\cosh (c + dx^n))^p \, dx = \int (ex)^m (a + b\cosh (c + dx^n))^p \, dx$$

input
```
integrate((e*x)**m*(a+b*cosh(c+d*x**n))**p,x)
```

output
```
Integral((e*x)**m*(a + b*cosh(c + d*x**n))**p, x)
```

3.42.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\cosh (c + dx^n))^p \, dx = \int (ex)^m (b\cosh (dx^n + c) + a)^p \, dx$$

input
```
integrate((e*x)^m*(a+b*cosh(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*(b*cosh(d*x^n + c) + a)^p, x)
```

3.42.8 Giac [N/A]

Not integrable

Time = 7.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\cosh(c + dx^n))^p \, dx = \int (ex)^m (b\cosh(dx^n + c) + a)^p \, dx$$

input
```
integrate((e*x)^m*(a+b*cosh(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*(b*cosh(d*x^n + c) + a)^p, x)
```

3.42.9 Mupad [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\cosh(c + dx^n))^p \, dx = \int (e\,x)^m (a + b\cosh(c + d\,x^n))^p \, dx$$

input
```
int((e*x)^m*(a + b*cosh(c + d*x^n))^p,x)
```

output
```
int((e*x)^m*(a + b*cosh(c + d*x^n))^p, x)
```

3.43 $\int (ex)^{-1+n} (b\cosh(c+dx^n))^p \, dx$

3.43.1 Optimal result

Integrand size = 20, antiderivative size = 95

$$\int (ex)^{-1+n} (b\cosh(c+dx^n))^p \, dx = -\frac{x^{-n}(ex)^n (b\cosh(c+dx^n))^{1+p} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \cosh^2(c+dx^n)\right) \sinh(c+dx^n)}{bden(1+p)\sqrt{-\sinh^2(c+dx^n)}}$$

output
```
-(e*x)^n*(b*cosh(c+d*x^n))^(p+1)*hypergeom([1/2, 1/2+1/2*p],[3/2+1/2*p],co
sh(c+d*x^n)^2)*sinh(c+d*x^n)/b/d/e/n/(p+1)/(x^n)/(-sinh(c+d*x^n)^2)^(1/2)
```

3.43.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\int (ex)^{-1+n} (b\cosh(c+dx^n))^p \, dx = \frac{x^{1-n}(ex)^{-1+n} (b\cosh(c+dx^n))^p \coth(c+dx^n) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \cosh^2(c+dx^n)\right) \sqrt{-\sinh^2(c+dx^n)}}{dn(1+p)}$$

input
```
Integrate[(e*x)^(-1 + n)*(b*Cosh[c + d*x^n])^p,x]
```

output
```
(x^(1 - n)*(e*x)^(-1 + n)*(b*Cosh[c + d*x^n])^p*Coth[c + d*x^n]*Hypergeome
tric2F1[1/2, (1 + p)/2, (3 + p)/2, Cosh[c + d*x^n]^2]*Sqrt[-Sinh[c + d*x^n
]^2])/(d*n*(1 + p))
```

3.43.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5846, 5844, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{n-1} (b\cosh(c+dx^n))^p \, dx$$

$$\Big\downarrow \text{ 5846}$$

$$\frac{x^{-n}(ex)^n \int x^{n-1}(b\cosh(dx^n+c))^p dx}{e}$$

$$\Big\downarrow \text{ 5844}$$

$$\frac{x^{-n}(ex)^n \int (b\cosh(dx^n+c))^p dx^n}{en}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x^{-n}(ex)^n \int \left(b\sin\left(idx^n+ic+\frac{\pi}{2}\right)\right)^p dx^n}{en}$$

$$\Big\downarrow \text{ 3122}$$

$$-\frac{x^{-n}(ex)^n \sinh(c+dx^n) (b\cosh(c+dx^n))^{p+1} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{p+1}{2},\frac{p+3}{2},\cosh^2(dx^n+c)\right)}{bden(p+1)\sqrt{-\sinh^2(c+dx^n)}}$$

input
```
Int[(e*x)^(-1 + n)*(b*Cosh[c + d*x^n])^p,x]
```

output
```
-(((e*x)^n*(b*Cosh[c + d*x^n])^(1 + p)*Hypergeometric2F1[1/2, (1 + p)/2, (
3 + p)/2, Cosh[c + d*x^n]^2]*Sinh[c + d*x^n])/(b*d*e*n*(1 + p)*x^n*Sqrt[-S
inh[c + d*x^n]^2]))
```

3.43.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

rule 5846
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_), x
_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && I
ntegerQ[Simplify[(m + 1)/n]]
```

3.43.4 Maple [F]

$$\int (ex)^{-1+n} (b \cosh (c + d x^n))^p dx$$

input
```
int((e*x)^(-1+n)*(b*cosh(c+d*x^n))^p,x)
```

output
```
int((e*x)^(-1+n)*(b*cosh(c+d*x^n))^p,x)
```

3.43.5 Fricas [F]

$$\int (ex)^{-1+n}(b\cosh(c+dx^n))^p\,dx=\int (ex)^{n-1}(b\cosh(dx^n+c))^p\,dx$$

input
```
integrate((e*x)^(-1+n)*(b*cosh(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^(n - 1)*(b*cosh(d*x^n + c))^p, x)
```

3.43.6 Sympy [F]

$$\int (ex)^{-1+n}(b\cosh(c+dx^n))^p\,dx=\int (b\cosh(c+dx^n))^p\,(ex)^{n-1}\,dx$$

input
```
integrate((e*x)**(-1+n)*(b*cosh(c+d*x**n))**p,x)
```

output
```
Integral((b*cosh(c + d*x**n))**p*(e*x)**(n - 1), x)
```

3.43.7 Maxima [F]

$$\int (ex)^{-1+n}(b\cosh(c+dx^n))^p\,dx=\int (ex)^{n-1}(b\cosh(dx^n+c))^p\,dx$$

input
```
integrate((e*x)^(-1+n)*(b*cosh(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(n - 1)*(b*cosh(d*x^n + c))^p, x)
```

3.43.8 Giac [F]

$$\int (ex)^{-1+n} (b\cosh (c+dx^n))^p \, dx = \int (ex)^{n-1} (b\cosh (dx^n+c))^p \, dx$$

input
```
integrate((e*x)^(-1+n)*(b*cosh(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(n - 1)*(b*cosh(d*x^n + c))^p, x)
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n} (b\cosh (c+dx^n))^p \, dx = \int (e\,x)^{n-1} (b\cosh(c+d\,x^n))^p \, dx$$

input
```
int((e*x)^(n - 1)*(b*cosh(c + d*x^n))^p,x)
```

output
```
int((e*x)^(n - 1)*(b*cosh(c + d*x^n))^p, x)
```

3.44 $\int (ex)^{-1+2n} (b \cosh (c+dx^n))^p \, dx$

3.44.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (ex)^{-1+2n} (b \cosh (c+dx^n))^p \, dx = \frac{x^{-2n} (ex)^{2n} \text{Int}(x^{-1+2n} (b \cosh (c+dx^n))^p, x)}{e}$$

output
```
(e*x)^(2*n)*Unintegrable(x^(-1+2*n)*(b*cosh(c+d*x^n))^p,x)/e/(x^(2*n))
```

3.44.2 Mathematica [N/A]

Not integrable

Time = 4.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n} (b \cosh (c+dx^n))^p \, dx = \int (ex)^{-1+2n} (b \cosh (c+dx^n))^p \, dx$$

input
```
Integrate[(e*x)^(-1 + 2*n)*(b*Cosh[c + d*x^n])^p,x]
```

output
```
Integrate[(e*x)^(-1 + 2*n)*(b*Cosh[c + d*x^n])^p, x]
```

3.44.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5846, 5890}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1} (b\cosh (c+dx^n))^p \, dx$$

$$\Big\downarrow \ 5846$$

$$\frac{x^{-2n}(ex)^{2n} \int x^{2n-1}(b\cosh (dx^n+c))^p dx}{e}$$

$$\Big\downarrow \ 5890$$

$$\frac{x^{-2n}(ex)^{2n} \int x^{2n-1}(b\cosh (dx^n+c))^p dx}{e}$$

input
```
Int[(e*x)^(-1 + 2*n)*(b*Cosh[c + d*x^n])^p,x]
```

output
```
$Aborted
```

3.44.3.1 Defintions of rubi rules used

rule 5846
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_), x
_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && I
ntegerQ[Simplify[(m + 1)/n]]
```

rule 5890
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_))^(m_.),
 x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Cosh[c + d*u^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && LinearQ[u, x]
```

3.44.4 Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (ex)^{2n-1} \left(b\cosh \left(c+d\,x^{n}\right)\right)^{p}dx$$

input
```
int((e*x)^(2*n-1)*(b*cosh(c+d*x^n))^p,x)
```

output
```
int((e*x)^(2*n-1)*(b*cosh(c+d*x^n))^p,x)
```

3.44.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n} (b\cosh (c+dx^n))^p \, dx = \int (ex)^{2\,n-1} (b\cosh (dx^n+c))^p \, dx$$

input
```
integrate((e*x)^(-1+2*n)*(b*cosh(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^(2*n - 1)*(b*cosh(d*x^n + c))^p, x)
```

3.44.6 Sympy [N/A]

Not integrable

Time = 7.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int (ex)^{-1+2n} (b\cosh (c+dx^n))^p \, dx = \int (b\cosh (c+dx^n))^p (ex)^{2n-1} \, dx$$

input
```
integrate((e*x)**(-1+2*n)*(b*cosh(c+d*x**n))**p,x)
```

output
```
Integral((b*cosh(c + d*x**n))**p*(e*x)**(2*n - 1), x)
```

3.44.7 Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n} (b\cosh(c+dx^n))^p \, dx = \int (ex)^{2n-1} (b\cosh(dx^n+c))^p \, dx$$

input
```
integrate((e*x)^(-1+2*n)*(b*cosh(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(2*n - 1)*(b*cosh(d*x^n + c))^p, x)
```

3.44.8 Giac [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n} (b\cosh(c+dx^n))^p \, dx = \int (ex)^{2n-1} (b\cosh(dx^n+c))^p \, dx$$

input
```
integrate((e*x)^(-1+2*n)*(b*cosh(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(2*n - 1)*(b*cosh(d*x^n + c))^p, x)
```

3.44.9 Mupad [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n} (b\cosh(c+dx^n))^p \, dx = \int (e\,x)^{2\,n-1} \left(b\cosh(c+d\,x^n)\right)^p dx$$

input
```
int((e*x)^(2*n - 1)*(b*cosh(c + d*x^n))^p,x)
```

output
```
int((e*x)^(2*n - 1)*(b*cosh(c + d*x^n))^p, x)
```

3.45 $\int (ex)^{-1+n} (a+b\cosh(c+dx^n))^p \, dx$

3.45.1 Optimal result

Integrand size = 22, antiderivative size = 131

$$\int (ex)^{-1+n} (a+b\cosh(c+dx^n))^p \, dx$$
$$= \frac{\sqrt{2}x^{-n}(ex)^n \operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},\frac{1}{2}(1-\cosh(c+dx^n)),\frac{b(1-\cosh(c+dx^n))}{a+b}\right)(a+b\cosh(c+dx^n))^p \left(\frac{a+b\cos}{a}\right.}{den\sqrt{1+\cosh(c+dx^n)}}$$

output
```
(e*x)^n*AppellF1(1/2,-p,1/2,3/2,b*(1-cosh(c+d*x^n))/(a+b),1/2-1/2*cosh(c+d
*x^n))*(a+b*cosh(c+d*x^n))^p*sinh(c+d*x^n)*2^(1/2)/d/e/n/(x^n)/(((a+b*cosh
(c+d*x^n))/(a+b))^p)/(1+cosh(c+d*x^n))^(1/2)
```

3.45.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.13

$$\int (ex)^{-1+n} (a+b\cosh(c+dx^n))^p \, dx$$
$$= \frac{x^{-n}(ex)^n \operatorname{AppellF1}\left(1+p,\frac{1}{2},\frac{1}{2},2+p,\frac{a+b\cosh(c+dx^n)}{a+b},\frac{a+b\cosh(c+dx^n)}{a-b}\right)\sqrt{-\frac{b(-1+\cosh(c+dx^n))}{a+b}}\sqrt{\frac{b(1+\cosh(c+dx^n))}{-a+b}}\left(\right.}{bden(1+p)}$$

input
```
Integrate[(e*x)^(-1 + n)*(a + b*Cosh[c + d*x^n])^p,x]
```

output
```
((e*x)^n*AppellF1[1 + p, 1/2, 1/2, 2 + p, (a + b*Cosh[c + d*x^n])/(a + b),
 (a + b*Cosh[c + d*x^n])/(a - b)]*Sqrt[-((b*(-1 + Cosh[c + d*x^n]))/(a + b
))]*Sqrt[(b*(1 + Cosh[c + d*x^n]))/(-a + b)]*(a + b*Cosh[c + d*x^n])^(1 +
p)*Csch[c + d*x^n])/(b*d*e*n*(1 + p)*x^n)
```

3.45.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5846, 5844, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{n-1} (a+b\cosh(c+dx^n))^p \, dx$$

$$\Big\downarrow \text{ 5846}$$

$$\frac{x^{-n}(ex)^n\int x^{n-1}(a+b\cosh(dx^n+c))^pdx}{e}$$

$$\Big\downarrow \text{ 5844}$$

$$\frac{x^{-n}(ex)^n\int (a+b\cosh(dx^n+c))^pdx^n}{en}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x^{-n}(ex)^n\int \left(a+b\sin\left(idx^n+ic+\frac{\pi}{2}\right)\right)^pdx^n}{en}$$

$$\Big\downarrow \text{ 3144}$$

$$-\frac{x^{-n}(ex)^n\sinh(c+dx^n)\int \frac{(a+b\cosh(dx^n+c))^p}{\sqrt{1-\cosh(dx^n+c)}\sqrt{\cosh(dx^n+c)+1}}d\cosh(dx^n+c)}{den\sqrt{1-\cosh(c+dx^n)}\sqrt{\cosh(c+dx^n)+1}}$$

$$\Big\downarrow \text{ 156}$$

$$-\frac{x^{-n}(ex)^n\sinh(c+dx^n)(a+b\cosh(c+dx^n))^p\left(\frac{a+b\cosh(c+dx^n)}{a+b}\right)^{-p}\int \frac{\left(\frac{a}{a+b}+\frac{b\cosh(dx^n+c)}{a+b}\right)^p}{\sqrt{1-\cosh(dx^n+c)}\sqrt{\cosh(dx^n+c)+1}}d\cosh(dx^n+}{den\sqrt{1-\cosh(c+dx^n)}\sqrt{\cosh(c+dx^n)+1}}$$

$$\Big\downarrow \text{ 155}$$

$$\frac{\sqrt{2}x^{-n}(ex)^n \sinh(c+dx^n)(a+b\cosh(c+dx^n))^p\left(\frac{a+b\cosh(c+dx^n)}{a+b}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},\frac{1}{2}(1-\cosh(dx^n+c$$
$$den\sqrt{\cosh(c+dx^n)+1}$$

input
```
Int[(e*x)^(-1 + n)*(a + b*Cosh[c + d*x^n])^p,x]
```

output
```
(Sqrt[2]*(e*x)^n*AppellF1[1/2, 1/2, -p, 3/2, (1 - Cosh[c + d*x^n])/2, (b*(
1 - Cosh[c + d*x^n]))/(a + b)]*(a + b*Cosh[c + d*x^n])^p*Sinh[c + d*x^n])/
(d*e*n*x^n*Sqrt[1 + Cosh[c + d*x^n]]*((a + b*Cosh[c + d*x^n])/(a + b))^p)
```

3.45.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

rule 5846
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_), x
_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && I
ntegerQ[Simplify[(m + 1)/n]]
```

3.45.4 Maple [F]

$$\int (ex)^{-1+n} \left(a+b\cosh \left(c+d\,x^{n}\right)\right)^{p}dx$$

input
```
int((e*x)^(-1+n)*(a+b*cosh(c+d*x^n))^p,x)
```

output
```
int((e*x)^(-1+n)*(a+b*cosh(c+d*x^n))^p,x)
```

3.45.5 Fricas [F]

$$\int (ex)^{-1+n}\left(a+b\cosh \left(c+dx^{n}\right)\right)^{p}\,dx=\int (ex)^{n-1}\left(b\cosh \left(dx^{n}+c\right)+a\right)^{p}dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^(n - 1)*(b*cosh(d*x^n + c) + a)^p, x)
```

3.45.6 Sympy [F]

$$\int (ex)^{-1+n}(a+b\cosh(c+dx^n))^p\,dx=\int (ex)^{n-1}(a+b\cosh(c+dx^n))^p\,dx$$

input
```
integrate((e*x)**(-1+n)*(a+b*cosh(c+d*x**n))**p,x)
```

output
```
Integral((e*x)**(n - 1)*(a + b*cosh(c + d*x**n))**p, x)
```

3.45.7 Maxima [F]

$$\int (ex)^{-1+n}(a+b\cosh(c+dx^n))^p\,dx=\int (ex)^{n-1}(b\cosh(dx^n+c)+a)^p\,dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(n - 1)*(b*cosh(d*x^n + c) + a)^p, x)
```

3.45.8 Giac [F]

$$\int (ex)^{-1+n}(a+b\cosh(c+dx^n))^p\,dx=\int (ex)^{n-1}(b\cosh(dx^n+c)+a)^p\,dx$$

input
```
integrate((e*x)^(-1+n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(n - 1)*(b*cosh(d*x^n + c) + a)^p, x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n} (a + b\cosh(c + dx^n))^p \, dx = \int (e\,x)^{n-1} (a + b\cosh(c + d\,x^n))^p \, dx$$

input
```
int((e*x)^(n - 1)*(a + b*cosh(c + d*x^n))^p,x)
```

output
```
int((e*x)^(n - 1)*(a + b*cosh(c + d*x^n))^p, x)
```

3.46 $\int (ex)^{-1+2n} (a + b\cosh(c + dx^n))^p \, dx$

3.46.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (ex)^{-1+2n} (a + b\cosh(c + dx^n))^p \, dx = \frac{x^{-2n}(ex)^{2n}\text{Int}(x^{-1+2n}(a + b\cosh(c + dx^n))^p, x)}{e}$$

output
```
(e*x)^(2*n)*Unintegrable(x^(-1+2*n)*(a+b*cosh(c+d*x^n))^p,x)/e/(x^(2*n))
```

3.46.2 Mathematica [N/A]

Not integrable

Time = 6.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n} (a + b\cosh(c + dx^n))^p \, dx = \int (ex)^{-1+2n} (a + b\cosh(c + dx^n))^p \, dx$$

input
```
Integrate[(e*x)^(-1 + 2*n)*(a + b*Cosh[c + d*x^n])^p,x]
```

output
```
Integrate[(e*x)^(-1 + 2*n)*(a + b*Cosh[c + d*x^n])^p, x]
```

3.46.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5846, 5890}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1} (a + b\cosh(c + dx^n))^p \, dx$$

$$\Big\downarrow \text{5846}$$

$$\frac{x^{-2n}(ex)^{2n} \int x^{2n-1}(a + b\cosh(dx^n + c))^p \, dx}{e}$$

$$\Big\downarrow \text{5890}$$

$$\frac{x^{-2n}(ex)^{2n} \int x^{2n-1}(a + b\cosh(dx^n + c))^p \, dx}{e}$$

input
```
Int[(e*x)^(-1 + 2*n)*(a + b*Cosh[c + d*x^n])^p,x]
```

output
```
$Aborted
```

3.46.3.1 Defintions of rubi rules used

rule 5846
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_), x
_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(
a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && I
ntegerQ[Simplify[(m + 1)/n]]
```

rule 5890
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_))^(m_.),
 x_Symbol] :> Unintegrable[(e*x)^m*(a + b*Cosh[c + d*u^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && LinearQ[u, x]
```

3.46.4 Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (ex)^{2n-1} (a+b\cosh (c+d\,x^{n}))^{p} dx$$

input
```
int((e*x)^(2*n-1)*(a+b*cosh(c+d*x^n))^p,x)
```

output
```
int((e*x)^(2*n-1)*(a+b*cosh(c+d*x^n))^p,x)
```

3.46.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n} (a+b\cosh (c+dx^n))^p \, dx = \int (ex)^{2\,n-1} (b\cosh (dx^n+c)+a)^p \, dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^(2*n - 1)*(b*cosh(d*x^n + c) + a)^p, x)
```

3.46.6 Sympy [N/A]

Not integrable

Time = 24.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int (ex)^{-1+2n} (a+b\cosh (c+dx^n))^p \, dx = \int (ex)^{2n-1} (a+b\cosh (c+dx^n))^p \, dx$$

input
```
integrate((e*x)**(-1+2*n)*(a+b*cosh(c+d*x**n))**p,x)
```

output
```
Integral((e*x)**(2*n - 1)*(a + b*cosh(c + d*x**n))**p, x)
```

3.46.7 Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n}\,(a+b\cosh(c+dx^n))^p\,dx = \int (ex)^{2n-1}\,(b\cosh(dx^n+c)+a)^p\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^(2*n - 1)*(b*cosh(d*x^n + c) + a)^p, x)
```

3.46.8 Giac [N/A]

Not integrable

Time = 7.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n}\,(a+b\cosh(c+dx^n))^p\,dx = \int (ex)^{2n-1}\,(b\cosh(dx^n+c)+a)^p\,dx$$

input
```
integrate((e*x)^(-1+2*n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^(2*n - 1)*(b*cosh(d*x^n + c) + a)^p, x)
```

3.46.9 Mupad [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n}\,(a+b\cosh(c+dx^n))^p\,dx = \int (e\,x)^{2n-1}\,(a+b\cosh(c+d\,x^n))^p\,dx$$

input
```
int((e*x)^(2*n - 1)*(a + b*cosh(c + d*x^n))^p,x)
```

output
```
int((e*x)^(2*n - 1)*(a + b*cosh(c + d*x^n))^p, x)
```

3.47 $\int x^m \cosh(a + bx^n)\, dx$

3.47.1 Optimal result

Integrand size = 12, antiderivative size = 89

$$\int x^m \cosh(a+bx^n)\, dx = -\frac{e^a x^{1+m}(-bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n}, -bx^n\right)}{2n} - \frac{e^{-a} x^{1+m}(bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n}, bx^n\right)}{2n}$$

output
```
-1/2*exp(a)*x^(1+m)*GAMMA((1+m)/n,-b*x^n)/n/((-b*x^n)^((1+m)/n))-1/2*x^(1+
m)*GAMMA((1+m)/n,b*x^n)/exp(a)/n/((b*x^n)^((1+m)/n))
```

3.47.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int x^m \cosh(a+bx^n)\, dx = -\frac{e^a x^{1+m}(-bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n}, -bx^n\right) + e^{-a} x^{1+m}(bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n}, bx^n\right)}{2n}$$

input
```
Integrate[x^m*Cosh[a + b*x^n],x]
```

output
```
-1/2*((E^a*x^(1 + m)*Gamma[(1 + m)/n, -(b*x^n)])/(-(b*x^n))^((1 + m)/n) +
(x^(1 + m)*Gamma[(1 + m)/n, b*x^n])/(E^a*(b*x^n)^((1 + m)/n)))/n
```

3.47.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5884, 2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cosh \left(a+bx^n\right) \, dx$$

$$\Big\downarrow \text{5884}$$

$$\frac{1}{2}\int e^{-bx^n-a}x^m dx+\frac{1}{2}\int e^{bx^n+a}x^m dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{e^a x^{m+1} (-bx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n},-bx^n\right)}{2n}-\frac{e^{-a} x^{m+1} (bx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n},bx^n\right)}{2n}$$

input
```
Int[x^m*Cosh[a + b*x^n],x]
```

output
```
-1/2*(E^a*x^(1 + m)*Gamma[(1 + m)/n, -(b*x^n)])/(n*(-(b*x^n))^((1 + m)/n))
 - (x^(1 + m)*Gamma[(1 + m)/n, b*x^n])/(2*E^a*n*(b*x^n)^((1 + m)/n))
```

3.47.3.1 Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

rule 5884
```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[1/2
   Int[(e*x)^m*E^(c + d*x^n), x], x] + Simp[1/2   Int[(e*x)^m*E^(-c - d*x^n
), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

3.47.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

method	result
meijerg	$\frac{x^{1+m} \operatorname{hypergeom}\left(\left[\frac{m}{2n}+\frac{1}{2n}\right],\left[\frac{1}{2},1+\frac{m}{2n}+\frac{1}{2n}\right],\frac{x^{2n} b^2}{4}\right) \cosh(a)}{1+m}+\frac{x^{n+m+1} b \operatorname{hypergeom}\left(\left[\frac{1}{2}+\frac{m}{2n}+\frac{1}{2n}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{m}{2n}+\frac{1}{2n}\right],\frac{x^{2n} b^2}{4}\right) \sinh(a)}{n+m+1}$

input
```
int(x^m*cosh(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/(1+m)*x^(1+m)*hypergeom([1/2/n*m+1/2/n],[1/2,1+1/2/n*m+1/2/n],1/4*x^(2*n
)*b^2)*cosh(a)+1/(n+m+1)*x^(n+m+1)*b*hypergeom([1/2+1/2/n*m+1/2/n],[3/2,3/
2+1/2/n*m+1/2/n],1/4*x^(2*n)*b^2)*sinh(a)
```

3.47.5 Fricas [F]

$$\int x^m \cosh(a+bx^n)\ dx=\int x^m \cosh(bx^n+a)\ dx$$

input
```
integrate(x^m*cosh(a+b*x^n),x, algorithm="fricas")
```

output
```
integral(x^m*cosh(b*x^n + a), x)
```

3.47.6 Sympy [F]

$$\int x^m \cosh(a+bx^n)\ dx=\int x^m \cosh(a+bx^n)\, dx$$

input
```
integrate(x**m*cosh(a+b*x**n),x)
```

output
```
Integral(x**m*cosh(a + b*x**n), x)
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int x^m \cosh(a+bx^n)\,dx = -\frac{x^{m+1}e^{(-a)}\Gamma\left(\frac{m+1}{n},bx^n\right)}{2\,(bx^n)^{\frac{m+1}{n}}\,n} - \frac{x^{m+1}e^{a}\Gamma\left(\frac{m+1}{n},-bx^n\right)}{2\,(-bx^n)^{\frac{m+1}{n}}\,n}$$

input
```
integrate(x^m*cosh(a+b*x^n),x, algorithm="maxima")
```

output
```
-1/2*x^(m + 1)*e^(-a)*gamma((m + 1)/n, b*x^n)/((b*x^n)^((m + 1)/n)*n) - 1/
2*x^(m + 1)*e^a*gamma((m + 1)/n, -b*x^n)/((-b*x^n)^((m + 1)/n)*n)
```

3.47.8 Giac [F]

$$\int x^m \cosh(a+bx^n)\,dx = \int x^m \cosh(bx^n+a)\,dx$$

input
```
integrate(x^m*cosh(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x^n + a), x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int x^m \cosh(a+bx^n)\,dx = \int x^m \cosh(a+b\,x^n)\,dx$$

input
```
int(x^m*cosh(a + b*x^n),x)
```

output
```
int(x^m*cosh(a + b*x^n), x)
```

3.48 $\int x^m \cosh^2(a+bx^n)\,dx$

3.48.1 Optimal result

Integrand size = 14, antiderivative size = 128

$$\int x^m \cosh^2(a+bx^n)\,dx = \frac{x^{1+m}}{2(1+m)} - \frac{2^{-\frac{1+m+2n}{n}} e^{2a} x^{1+m} (-bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -2bx^n\right)}{n} - \frac{2^{-\frac{1+m+2n}{n}} e^{-2a} x^{1+m} (bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, 2bx^n\right)}{n}$$

output
```
1/2*x^(1+m)/(1+m)-exp(2*a)*x^(1+m)*GAMMA((1+m)/n,-2*b*x^n)/(2^((1+m+2*n)/n
))/n/((-b*x^n)^((1+m)/n))-x^(1+m)*GAMMA((1+m)/n,2*b*x^n)/(2^((1+m+2*n)/n))
/exp(2*a)/n/((b*x^n)^((1+m)/n))
```

3.48.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

$$\int x^m \cosh^2(a+bx^n)\,dx = -\frac{x^{1+m}\left(-2n + 2^{-\frac{1+m}{n}} e^{2a}(1+m)(-bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -2bx^n\right) + 2^{-\frac{1+m}{n}} e^{-2a}(1+m)(bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, 2b\right.}{4(1+m)n}$$

input
```
Integrate[x^m*Cosh[a + b*x^n]^2,x]
```

output
```
-1/4*(x^(1 + m)*(-2*n + (E^(2*a)*(1 + m)*Gamma[(1 + m)/n, -2*b*x^n])/(2^((
1 + m)/n)*(-(b*x^n))^((1 + m)/n)) + ((1 + m)*Gamma[(1 + m)/n, 2*b*x^n])/(2
^((1 + m)/n)*E^(2*a)*(b*x^n)^((1 + m)/n))))/((1 + m)*n)
```

3.48.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5886, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cosh^2\left(a+bx^n\right)dx$$

$$\Big\downarrow \text{5886}$$

$$\int \left(\frac{1}{2}x^m \cosh\left(2a+2bx^n\right)+\frac{x^m}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}2^{-\frac{m+2n+1}{n}}x^{m+1}(-bx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-2bx^n\right)}{n}-\frac{e^{-2a}2^{-\frac{m+2n+1}{n}}x^{m+1}(bx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},2bx^n\right)}{n}+\frac{x^{m+1}}{2(m+1)}$$

input
```
Int[x^m*Cosh[a + b*x^n]^2,x]
```

output
```
x^(1 + m)/(2*(1 + m)) - (E^(2*a)*x^(1 + m)*Gamma[(1 + m)/n, -2*b*x^n])/(2^
((1 + m + 2*n)/n)*n*(-(b*x^n))^((1 + m)/n)) - (x^(1 + m)*Gamma[(1 + m)/n,
2*b*x^n])/(2^((1 + m + 2*n)/n)*E^(2*a)*n*(b*x^n)^((1 + m)/n))
```

3.48.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5886
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

3.48.4 Maple [F]

$$\int x^m \cosh\left(a + b\,x^n\right)^2 dx$$

input
```
int(x^m*cosh(a+b*x^n)^2,x)
```

output
```
int(x^m*cosh(a+b*x^n)^2,x)
```

3.48.5 Fricas [F]

$$\int x^m \cosh^2\left(a + bx^n\right)\, dx = \int x^m \cosh\left(bx^n + a\right)^2\, dx$$

input
```
integrate(x^m*cosh(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral(x^m*cosh(b*x^n + a)^2, x)
```

3.48.6 Sympy [F]

$$\int x^m \cosh^2\left(a + bx^n\right)\, dx = \int x^m \cosh^2\left(a + bx^n\right) dx$$

input
```
integrate(x**m*cosh(a+b*x**n)**2,x)
```

output
```
Integral(x**m*cosh(a + b*x**n)**2, x)
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.79

$$\int x^m \cosh^2\left(a+bx^n\right) dx$$
$$= -\frac{x^{m+1}e^{(-2a)}\Gamma\left(\frac{m+1}{n}, 2bx^n\right)}{4\left(2bx^n\right)^{\frac{m+1}{n}}n} - \frac{x^{m+1}e^{(2a)}\Gamma\left(\frac{m+1}{n}, -2bx^n\right)}{4\left(-2bx^n\right)^{\frac{m+1}{n}}n} + \frac{x^{m+1}}{2(m+1)}$$

input
```
integrate(x^m*cosh(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-1/4*x^(m + 1)*e^(-2*a)*gamma((m + 1)/n, 2*b*x^n)/((2*b*x^n)^((m + 1)/n)*n
) - 1/4*x^(m + 1)*e^(2*a)*gamma((m + 1)/n, -2*b*x^n)/((-2*b*x^n)^((m + 1)/
n)*n) + 1/2*x^(m + 1)/(m + 1)
```

3.48.8 Giac [F]

$$\int x^m \cosh^2\left(a+bx^n\right) dx = \int x^m \cosh\left(bx^n+a\right)^2 dx$$

input
```
integrate(x^m*cosh(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x^n + a)^2, x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int x^m \cosh^2\left(a+bx^n\right) dx = \int x^m \cosh(a+b\,x^n)^2\, dx$$

input
```
int(x^m*cosh(a + b*x^n)^2,x)
```

output
```
int(x^m*cosh(a + b*x^n)^2, x)
```

3.49 $\int x^m \cosh^3(a+bx^n)\,dx$

3.49.1 Optimal result

Integrand size = 14, antiderivative size = 200

$$\int x^m \cosh^3(a+bx^n)\,dx = -\frac{3^{-\frac{1+m}{n}} e^{3a} x^{1+m} (-bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -3bx^n\right)}{8n} - \frac{3e^a x^{1+m} (-bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -bx^n\right)}{8n} - \frac{3e^{-a} x^{1+m} (bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, bx^n\right)}{8n} - \frac{3^{-\frac{1+m}{n}} e^{-3a} x^{1+m} (bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, 3bx^n\right)}{8n}$$

output
```
-1/8*exp(3*a)*x^(1+m)*GAMMA((1+m)/n,-3*b*x^n)/(3^((1+m)/n))/n/((-b*x^n)^((
1+m)/n))-3/8*exp(a)*x^(1+m)*GAMMA((1+m)/n,-b*x^n)/n/((-b*x^n)^((1+m)/n))-3
/8*x^(1+m)*GAMMA((1+m)/n,b*x^n)/exp(a)/n/((b*x^n)^((1+m)/n))-1/8*x^(1+m)*G
AMMA((1+m)/n,3*b*x^n)/(3^((1+m)/n))/exp(3*a)/n/((b*x^n)^((1+m)/n))
```

3.49.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91

$$\int x^m \cosh^3\left(a+bx^n\right)\,dx =$$

$$-\frac{3^{-\frac{1+m}{n}}e^{-3a}x^{1+m}\left(-b^2x^{2n}\right)^{-\frac{1+m}{n}}\left(e^{6a}(bx^n)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-3bx^n\right)+3^{\frac{1+m+n}{n}}e^{4a}(bx^n)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-bx^n\right)+(-b\right.}{8n}$$

input

```
Integrate[x^m*Cosh[a + b*x^n]^3,x]
```

output

```
-1/8*(x^(1 + m)*(E^(6*a)*(b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, -3*b*x^n] +
3^((1 + m + n)/n)*E^(4*a)*(b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, -(b*x^n)] +
 (-(b*x^n))^((1 + m)/n)*(3^((1 + m + n)/n)*E^(2*a)*Gamma[(1 + m)/n, b*x^n]
 + Gamma[(1 + m)/n, 3*b*x^n])))/(3^((1 + m)/n)*E^(3*a)*n*(-(b^2*x^(2*n)))^
((1 + m)/n))
```

3.49.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5886, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cosh^3\left(a+bx^n\right)\,dx$$

$$\Big\downarrow \text{5886}$$

$$\int \left(\frac{3}{4}x^m\cosh\left(a+bx^n\right)+\frac{1}{4}x^m\cosh\left(3a+3bx^n\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{3a}3^{-\frac{m+1}{n}}x^{m+1}(-bx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-3bx^n\right)}{8n}-\frac{3e^ax^{m+1}(-bx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x^{m+1}(bx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},bx^n\right)}{8n}-\frac{e^{-3a}3^{-\frac{m+1}{n}}x^{m+1}(bx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},3bx^n\right)}{8n}$$

input

```
Int[x^m*Cosh[a + b*x^n]^3,x]
```

output
```
-1/8*(E^(3*a)*x^(1 + m)*Gamma[(1 + m)/n, -3*b*x^n])/(3^((1 + m)/n)*n*(-(b*
x^n))^((1 + m)/n)) - (3*E^a*x^(1 + m)*Gamma[(1 + m)/n, -(b*x^n)])/(8*n*(-(
b*x^n))^((1 + m)/n)) - (3*x^(1 + m)*Gamma[(1 + m)/n, b*x^n])/(8*E^a*n*(b*x
^n)^((1 + m)/n)) - (x^(1 + m)*Gamma[(1 + m)/n, 3*b*x^n])/(8*3^((1 + m)/n)*
E^(3*a)*n*(b*x^n)^((1 + m)/n))
```

3.49.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5886
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

3.49.4 Maple [F]

$$\int x^m \cosh\left(a+b\,x^n\right)^3 dx$$

input
```
int(x^m*cosh(a+b*x^n)^3,x)
```

output
```
int(x^m*cosh(a+b*x^n)^3,x)
```

3.49.5 Fricas [F]

$$\int x^m \cosh^3\left(a+bx^n\right)\,dx=\int x^m \cosh\left(bx^n+a\right)^3\,dx$$

input
```
integrate(x^m*cosh(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral(x^m*cosh(b*x^n + a)^3, x)
```

3.49.6 Sympy [F]

$$\int x^m \cosh^3(a+bx^n)\,dx = \int x^m \cosh^3(a+bx^n)\,dx$$

input
```
integrate(x**m*cosh(a+b*x**n)**3,x)
```

output
```
Integral(x**m*cosh(a + b*x**n)**3, x)
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.86

$$\int x^m \cosh^3(a+bx^n)\,dx = -\frac{x^{m+1}e^{(-3a)}\Gamma\left(\frac{m+1}{n},3\,bx^n\right)}{8\,(3\,bx^n)^{\frac{m+1}{n}}n} - \frac{3\,x^{m+1}e^{(-a)}\Gamma\left(\frac{m+1}{n},bx^n\right)}{8\,(bx^n)^{\frac{m+1}{n}}n} - \frac{3\,x^{m+1}e^{a}\Gamma\left(\frac{m+1}{n},-bx^n\right)}{8\,(-bx^n)^{\frac{m+1}{n}}n} - \frac{x^{m+1}e^{(3a)}\Gamma\left(\frac{m+1}{n},-3\,bx^n\right)}{8\,(-3\,bx^n)^{\frac{m+1}{n}}n}$$

input
```
integrate(x^m*cosh(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
-1/8*x^(m + 1)*e^(-3*a)*gamma((m + 1)/n, 3*b*x^n)/((3*b*x^n)^((m + 1)/n)*n
) - 3/8*x^(m + 1)*e^(-a)*gamma((m + 1)/n, b*x^n)/((b*x^n)^((m + 1)/n)*n) -
 3/8*x^(m + 1)*e^a*gamma((m + 1)/n, -b*x^n)/((-b*x^n)^((m + 1)/n)*n) - 1/8
*x^(m + 1)*e^(3*a)*gamma((m + 1)/n, -3*b*x^n)/((-3*b*x^n)^((m + 1)/n)*n)
```

3.49.8 Giac [F]

$$\int x^m \cosh^3(a+bx^n)\,dx = \int x^m \cosh(bx^n+a)^3\,dx$$

input
```
integrate(x^m*cosh(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x^n + a)^3, x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int x^m \cosh^3\left(a+bx^n\right)\,dx=\int x^m\cosh(a+b\,x^n)^3\,dx$$

input
```
int(x^m*cosh(a + b*x^n)^3,x)
```

output
```
int(x^m*cosh(a + b*x^n)^3, x)
```

3.50 $\int x^{-1-n} \cosh(a+bx^n)\,dx$

3.50.1 Optimal result

Integrand size = 16, antiderivative size = 45

$$\int x^{-1-n} \cosh(a+bx^n)\,dx = -\frac{x^{-n}\cosh(a+bx^n)}{n} + \frac{b\text{Chi}(bx^n)\sinh(a)}{n} + \frac{b\cosh(a)\text{Shi}(bx^n)}{n}$$

output
```
-cosh(a+b*x^n)/n/(x^n)+b*cosh(a)*Shi(b*x^n)/n+b*Chi(b*x^n)*sinh(a)/n
```

3.50.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int x^{-1-n}\cosh(a+bx^n)\,dx = \frac{x^{-n}(-\cosh(a+bx^n) + bx^n\text{Chi}(bx^n)\sinh(a) + bx^n\cosh(a)\text{Shi}(bx^n))}{n}$$

input
```
Integrate[x^(-1 - n)*Cosh[a + b*x^n],x]
```

output
```
(-Cosh[a + b*x^n] + b*x^n*CoshIntegral[b*x^n]*Sinh[a] + b*x^n*Cosh[a]*Sinh
Integral[b*x^n])/(n*x^n)
```

3.50.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5844, 3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}\cosh\left(a+bx^n\right)\,dx$$

$$\Big\downarrow \text{ 5844}$$

$$\frac{\int x^{-2n}\cosh\left(bx^n+a\right)dx^n}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int x^{-2n}\sin\left(ibx^n+ia+\frac{\pi}{2}\right)dx^n}{n}$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{-x^{-n}\cosh\left(a+bx^n\right)+ib\int -ix^{-n}\sinh\left(bx^n+a\right)dx^n}{n}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{b\int x^{-n}\sinh\left(bx^n+a\right)dx^n-x^{-n}\cosh\left(a+bx^n\right)}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-x^{-n}\cosh\left(a+bx^n\right)+b\int -ix^{-n}\sin\left(ibx^n+ia\right)dx^n}{n}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{x^{-n}\left(-\cosh\left(a+bx^n\right)\right)-ib\int x^{-n}\sin\left(ibx^n+ia\right)dx^n}{n}$$

$$\Big\downarrow \text{ 3784}$$

$$\frac{x^{-n}\left(-\cosh\left(a+bx^n\right)\right)-ib\left(i\sinh(a)\int x^{-n}\cosh\left(bx^n\right)dx^n+\cosh(a)\int ix^{-n}\sinh\left(bx^n\right)dx^n\right)}{n}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{x^{-n}\left(-\cosh\left(a+bx^n\right)\right)-ib\left(i\sinh(a)\int x^{-n}\cosh\left(bx^n\right)dx^n+i\cosh(a)\int x^{-n}\sinh\left(bx^n\right)dx^n\right)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^{-n}(-\cosh(a+bx^n)) - ib\left(i\sinh(a)\int x^{-n}\sin\left(ibx^n+\frac{\pi}{2}\right)dx^n + i\cosh(a)\int -ix^{-n}\sin(ibx^n)\,dx^n\right)}{n}$$

$$\Big\downarrow \text{26}$$

$$\frac{x^{-n}(-\cosh(a+bx^n)) - ib\left(i\sinh(a)\int x^{-n}\sin\left(ibx^n+\frac{\pi}{2}\right)dx^n + \cosh(a)\int x^{-n}\sin(ibx^n)\,dx^n\right)}{n}$$

$$\Big\downarrow \text{3779}$$

$$\frac{x^{-n}(-\cosh(a+bx^n)) - ib\left(i\sinh(a)\int x^{-n}\sin\left(ibx^n+\frac{\pi}{2}\right)dx^n + i\cosh(a)\text{Shi}(bx^n)\right)}{n}$$

$$\Big\downarrow \text{3782}$$

$$\frac{x^{-n}(-\cosh(a+bx^n)) - ib(i\sinh(a)\text{Chi}(bx^n) + i\cosh(a)\text{Shi}(bx^n))}{n}$$

input
```
Int[x^(-1 - n)*Cosh[a + b*x^n],x]
```

output
```
(-(Cosh[a + b*x^n]/x^n) - I*b*(I*CoshIntegral[b*x^n]*Sinh[a] + I*Cosh[a]*S
inhIntegral[b*x^n]))/n
```

3.50.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5844
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplif
y[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplif
y[(m + 1)/n], 0]))
```

3.50.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

method	result	size
risch	$-\frac{\left(-b\,\mathrm{e}^{-a}\,\mathrm{Ei}_{1}(b\,x^{n})x^{n}+b\,\mathrm{e}^{a}\,\mathrm{Ei}_{1}(-b\,x^{n})x^{n}+\mathrm{e}^{-a-b\,x^{n}}+\mathrm{e}^{a+b\,x^{n}}\right)x^{-n}}{2n}$	63

input
```
int(x^(-1-n)*cosh(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-b*exp(-a)*Ei(1,b*x^n)*x^n+b*exp(a)*Ei(1,-b*x^n)*x^n+exp(-a-b*x^n)+e
xp(a+b*x^n))/n/(x^n)
```

3.50.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(45) = 90.

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.11

$$\int x^{-1-n}\cosh\left(a+bx^n\right)\,dx$$
$$= \frac{((b\cosh(a)+b\sinh(a))\cosh(n\log(x))+(b\cosh(a)+b\sinh(a))\sinh(n\log(x)))\mathrm{Ei}(b\cosh(n\log(x))+ \dots}{}$$

input
```
integrate(x^(-1-n)*cosh(a+b*x^n),x, algorithm="fricas")
```

output
```
1/2*(((b*cosh(a) + b*sinh(a))*cosh(n*log(x)) + (b*cosh(a) + b*sinh(a))*sin
h(n*log(x)))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) - ((b*cosh(a) - b*sin
h(a))*cosh(n*log(x)) + (b*cosh(a) - b*sinh(a))*sinh(n*log(x)))*Ei(-b*cosh(
n*log(x)) - b*sinh(n*log(x))) - 2*cosh(b*cosh(n*log(x)) + b*sinh(n*log(x))
 + a))/(n*cosh(n*log(x)) + n*sinh(n*log(x)))
```

3.50.6 Sympy [F]

$$\int x^{-1-n}\cosh\left(a+bx^n\right)\,dx=\int x^{-n-1}\cosh\left(a+bx^n\right)dx$$

input
```
integrate(x**(-1-n)*cosh(a+b*x**n),x)
```

output
```
Integral(x**(-n - 1)*cosh(a + b*x**n), x)
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int x^{-1-n}\cosh\left(a+bx^n\right)\,dx=-\frac{be^{(-a)}\Gamma(-1,bx^n)}{2\,n}+\frac{be^{a}\Gamma(-1,-bx^n)}{2\,n}$$

input
```
integrate(x^(-1-n)*cosh(a+b*x^n),x, algorithm="maxima")
```

output
```
-1/2*b*e^(-a)*gamma(-1, b*x^n)/n + 1/2*b*e^a*gamma(-1, -b*x^n)/n
```

3.50.8 Giac [F]

$$\int x^{-1-n}\cosh\left(a+bx^n\right)\,dx=\int x^{-n-1}\cosh\left(bx^n+a\right)\,dx$$

input
```
integrate(x^(-1-n)*cosh(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)*cosh(b*x^n + a), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\cosh\left(a+bx^n\right)\,dx=\int\frac{\cosh(a+b\,x^n)}{x^{n+1}}\,dx$$

input
```
int(cosh(a + b*x^n)/x^(n + 1),x)
```

output
```
int(cosh(a + b*x^n)/x^(n + 1), x)
```

3.51 $\int x^{-1-n} \cosh^2(a+bx^n)\, dx$

3.51.1 Optimal result

Integrand size = 18, antiderivative size = 67

$$\int x^{-1-n} \cosh^2(a+bx^n)\, dx = -\frac{x^{-n}}{2n} - \frac{x^{-n}\cosh(2(a+bx^n))}{2n} + \frac{b\text{Chi}(2bx^n)\sinh(2a)}{n} + \frac{b\cosh(2a)\text{Shi}(2bx^n)}{n}$$

output
```
-1/2/n/(x^n)-1/2*cosh(2*a+2*b*x^n)/n/(x^n)+b*cosh(2*a)*Shi(2*b*x^n)/n+b*Ch
i(2*b*x^n)*sinh(2*a)/n
```

3.51.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int x^{-1-n} \cosh^2(a+bx^n)\, dx = \frac{x^{-n}\left(-\cosh^2(a+bx^n) + bx^n\text{Chi}(2bx^n)\sinh(2a) + bx^n\cosh(2a)\text{Shi}(2bx^n)\right)}{n}$$

input
```
Integrate[x^(-1 - n)*Cosh[a + b*x^n]^2,x]
```

output
```
(-Cosh[a + b*x^n]^2 + b*x^n*CoshIntegral[2*b*x^n]*Sinh[2*a] + b*x^n*Cosh[2
*a]*SinhIntegral[2*b*x^n])/(n*x^n)
```

3.51.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5886, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}\cosh^2\left(a+bx^n\right)dx$$

$$\Big\downarrow \text{5886}$$

$$\int \left(\frac{1}{2}x^{-n-1}\cosh\left(2a+2bx^n\right)+\frac{x^{-n-1}}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\sinh(2a)\text{Chi}(2bx^n)}{n}+\frac{b\cosh(2a)\text{Shi}(2bx^n)}{n}-\frac{x^{-n}\cosh\left(2(a+bx^n)\right)}{2n}-\frac{x^{-n}}{2n}$$

input
```
Int[x^(-1 - n)*Cosh[a + b*x^n]^2,x]
```

output
```
-1/2*1/(n*x^n) - Cosh[2*(a + b*x^n)]/(2*n*x^n) + (b*CoshIntegral[2*b*x^n]*
Sinh[2*a])/n + (b*Cosh[2*a]*SinhIntegral[2*b*x^n])/n
```

3.51.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5886
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

3.51.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

method	result	size
risch	$\frac{\left(2b\,\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2b\,x^n)x^n-2b\,\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2b\,x^n)x^n-\mathrm{e}^{-2a-2b\,x^n}-\mathrm{e}^{2a+2b\,x^n}-2\right)x^{-n}}{4n}$	75

input
```
int(x^(-1-n)*cosh(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*b*exp(-2*a)*Ei(1,2*b*x^n)*x^n-2*b*exp(2*a)*Ei(1,-2*b*x^n)*x^n-exp(-
2*a-2*b*x^n)-exp(2*a+2*b*x^n)-2)/(x^n)/n
```

3.51.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(64) = 128.

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.72

$$\int x^{-1-n}\cosh^2(a+bx^n)\,dx$$
$$= \frac{((b\cosh(2a)+b\sinh(2a))\cosh(n\log(x))+(b\cosh(2a)+b\sinh(2a))\sinh(n\log(x)))\mathrm{Ei}(2b\cosh(n\log}{}$$

input
```
integrate(x^(-1-n)*cosh(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
1/2*(((b*cosh(2*a) + b*sinh(2*a))*cosh(n*log(x)) + (b*cosh(2*a) + b*sinh(2
*a))*sinh(n*log(x)))*Ei(2*b*cosh(n*log(x)) + 2*b*sinh(n*log(x))) - ((b*cos
h(2*a) - b*sinh(2*a))*cosh(n*log(x)) + (b*cosh(2*a) - b*sinh(2*a))*sinh(n*
log(x)))*Ei(-2*b*cosh(n*log(x)) - 2*b*sinh(n*log(x))) - cosh(b*cosh(n*log(
x)) + b*sinh(n*log(x)) + a)^2 - sinh(b*cosh(n*log(x)) + b*sinh(n*log(x)) +
 a)^2 - 1)/(n*cosh(n*log(x)) + n*sinh(n*log(x)))
```

3.51.6 Sympy [F]

$$\int x^{-1-n}\cosh^2(a+bx^n)\,dx=\int x^{-n-1}\cosh^2(a+bx^n)\,dx$$

input
```
integrate(x**(-1-n)*cosh(a+b*x**n)**2,x)
```

output
```
Integral(x**(-n - 1)*cosh(a + b*x**n)**2, x)
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int x^{-1-n}\cosh^2(a+bx^n)\,dx=-\frac{be^{(-2a)}\Gamma(-1,2bx^n)}{2n}+\frac{be^{(2a)}\Gamma(-1,-2bx^n)}{2n}-\frac{1}{2nx^n}$$

input
```
integrate(x^(-1-n)*cosh(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-1/2*b*e^(-2*a)*gamma(-1, 2*b*x^n)/n + 1/2*b*e^(2*a)*gamma(-1, -2*b*x^n)/n
 - 1/2/(n*x^n)
```

3.51.8 Giac [F]

$$\int x^{-1-n}\cosh^2(a+bx^n)\,dx=\int x^{-n-1}\cosh(bx^n+a)^2\,dx$$

input
```
integrate(x^(-1-n)*cosh(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)*cosh(b*x^n + a)^2, x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\cosh^2\left(a+bx^n\right)\,dx=\int\frac{\cosh(a+b\,x^n)^2}{x^{n+1}}\,dx$$

input
```
int(cosh(a + b*x^n)^2/x^(n + 1),x)
```

output
```
int(cosh(a + b*x^n)^2/x^(n + 1), x)
```

3.52 $\int x^{-1-n} \cosh^3(a + bx^n) \, dx$

3.52.1 Optimal result

Integrand size = 18, antiderivative size = 113

$$\int x^{-1-n} \cosh^3(a + bx^n) \, dx = -\frac{3x^{-n}\cosh(a + bx^n)}{4n} - \frac{x^{-n}\cosh(3(a + bx^n))}{4n} + \frac{3b\text{Chi}(bx^n)\sinh(a)}{4n} + \frac{3b\text{Chi}(3bx^n)\sinh(3a)}{4n} + \frac{3b\cosh(a)\text{Shi}(bx^n)}{4n} + \frac{3b\cosh(3a)\text{Shi}(3bx^n)}{4n}$$

output
```
-3/4*cosh(a+b*x^n)/n/(x^n)-1/4*cosh(3*a+3*b*x^n)/n/(x^n)+3/4*b*cosh(a)*Shi
(b*x^n)/n+3/4*b*cosh(3*a)*Shi(3*b*x^n)/n+3/4*b*Chi(b*x^n)*sinh(a)/n+3/4*b*
Chi(3*b*x^n)*sinh(3*a)/n
```

3.52.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int x^{-1-n} \cosh^3(a + bx^n) \, dx = \frac{x^{-n}(-3\cosh(a + bx^n) - \cosh(3(a + bx^n)) + 3bx^n\text{Chi}(bx^n)\sinh(a) + 3bx^n\text{Chi}(3bx^n)\sinh(3a) + 3bx^n \cos}{4n}$$

input
```
Integrate[x^(-1 - n)*Cosh[a + b*x^n]^3,x]
```

output
```
(-3*Cosh[a + b*x^n] - Cosh[3*(a + b*x^n)] + 3*b*x^n*CoshIntegral[b*x^n]*Si
nh[a] + 3*b*x^n*CoshIntegral[3*b*x^n]*Sinh[3*a] + 3*b*x^n*Cosh[a]*SinhInte
gral[b*x^n] + 3*b*x^n*Cosh[3*a]*SinhIntegral[3*b*x^n])/(4*n*x^n)
```

3.52.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5886, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}\cosh^3\left(a+bx^n\right)\,dx$$

$$\Big\downarrow \text{5886}$$

$$\int \left(\frac{3}{4}x^{-n-1}\cosh\left(a+bx^n\right)+\frac{1}{4}x^{-n-1}\cosh\left(3a+3bx^n\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b\sinh(a)\text{Chi}(bx^n)}{4n}+\frac{3b\sinh(3a)\text{Chi}(3bx^n)}{4n}+\frac{3b\cosh(a)\text{Shi}(bx^n)}{4n}+\frac{3b\cosh(3a)\text{Shi}(3bx^n)}{4n}-\frac{3x^{-n}\cosh\left(a+bx^n\right)}{4n}-\frac{x^{-n}\cosh\left(3(a+bx^n)\right)}{4n}$$

input
```
Int[x^(-1 - n)*Cosh[a + b*x^n]^3,x]
```

output
```
(-3*Cosh[a + b*x^n])/(4*n*x^n) - Cosh[3*(a + b*x^n)]/(4*n*x^n) + (3*b*Cosh
Integral[b*x^n]*Sinh[a])/(4*n) + (3*b*CoshIntegral[3*b*x^n]*Sinh[3*a])/(4*
n) + (3*b*Cosh[a]*SinhIntegral[b*x^n])/(4*n) + (3*b*Cosh[3*a]*SinhIntegral
[3*b*x^n])/(4*n)
```

3.52.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5886
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

3.52.4 Maple [A] (verified)

Time = 7.61 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\left(3b\,\mathrm{e}^{a}\,\mathrm{Ei}_1(-b\,x^n)x^n-3b\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3b\,x^n)x^n-3b\,\mathrm{e}^{-a}\,\mathrm{Ei}_1(b\,x^n)x^n+3b\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3b\,x^n)x^n+\mathrm{e}^{-3a-3b\,x^n}+3\,\mathrm{e}^{-a-b\,x^n}+\mathrm{e}^{3a+3b\,x^n}+3\,\mathrm{e}}{8n}$

input
```
int(x^(-1-n)*cosh(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(3*b*exp(a)*Ei(1,-b*x^n)*x^n-3*b*exp(-3*a)*Ei(1,3*b*x^n)*x^n-3*b*exp(
-a)*Ei(1,b*x^n)*x^n+3*b*exp(3*a)*Ei(1,-3*b*x^n)*x^n+exp(-3*a-3*b*x^n)+3*ex
p(-a-b*x^n)+exp(3*a+3*b*x^n)+3*exp(a+b*x^n))/(x^n)/n
```

3.52.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(102) = 204.

Time = 0.26 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.83

$$\int x^{-1-n}\cosh^3\left(a+bx^n\right)\,dx =$$

$$-\frac{2\,\cosh\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a\right)^3+6\,\cosh\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a\right)\mathrm{s}}{}$$

input
```
integrate(x^(-1-n)*cosh(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
-1/8*(2*cosh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)^3 + 6*cosh(b*cosh(n*
log(x)) + b*sinh(n*log(x)) + a)*sinh(b*cosh(n*log(x)) + b*sinh(n*log(x)) +
 a)^2 - 3*((b*cosh(3*a) + b*sinh(3*a))*cosh(n*log(x)) + (b*cosh(3*a) + b*s
inh(3*a))*sinh(n*log(x)))*Ei(3*b*cosh(n*log(x)) + 3*b*sinh(n*log(x))) - 3*
((b*cosh(a) + b*sinh(a))*cosh(n*log(x)) + (b*cosh(a) + b*sinh(a))*sinh(n*l
og(x)))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) + 3*((b*cosh(a) - b*sinh(a
))*cosh(n*log(x)) + (b*cosh(a) - b*sinh(a))*sinh(n*log(x)))*Ei(-b*cosh(n*l
og(x)) - b*sinh(n*log(x))) + 3*((b*cosh(3*a) - b*sinh(3*a))*cosh(n*log(x))
 + (b*cosh(3*a) - b*sinh(3*a))*sinh(n*log(x)))*Ei(-3*b*cosh(n*log(x)) - 3*
b*sinh(n*log(x))) + 6*cosh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a))/(n*co
sh(n*log(x)) + n*sinh(n*log(x)))
```

3.52.6 Sympy [F]

$$\int x^{-1-n}\cosh^3(a+bx^n)\,dx = \int x^{-n-1}\cosh^3(a+bx^n)\,dx$$

input
```
integrate(x**(-1-n)*cosh(a+b*x**n)**3,x)
```

output
```
Integral(x**(-n - 1)*cosh(a + b*x**n)**3, x)
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62

$$\int x^{-1-n}\cosh^3(a+bx^n)\,dx = -\frac{3\,be^{(-3\,a)}\Gamma(-1,3\,bx^n)}{8\,n} - \frac{3\,be^{(-a)}\Gamma(-1,bx^n)}{8\,n} + \frac{3\,be^{a}\Gamma(-1,-bx^n)}{8\,n} + \frac{3\,be^{(3\,a)}\Gamma(-1,-3\,bx^n)}{8\,n}$$

input
```
integrate(x^(-1-n)*cosh(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
-3/8*b*e^(-3*a)*gamma(-1, 3*b*x^n)/n - 3/8*b*e^(-a)*gamma(-1, b*x^n)/n + 3
/8*b*e^a*gamma(-1, -b*x^n)/n + 3/8*b*e^(3*a)*gamma(-1, -3*b*x^n)/n
```

3.52.8 Giac [F]

$$\int x^{-1-n}\cosh^3(a+bx^n)\,dx = \int x^{-n-1}\cosh(bx^n+a)^3\,dx$$

input
```
integrate(x^(-1-n)*cosh(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)*cosh(b*x^n + a)^3, x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\cosh^3(a+bx^n)\,dx = \int \frac{\cosh(a+b\,x^n)^3}{x^{n+1}}\,dx$$

input
```
int(cosh(a + b*x^n)^3/x^(n + 1),x)
```

output
```
int(cosh(a + b*x^n)^3/x^(n + 1), x)
```

3.53 $\int x^{-1+\frac{n}{2}} \cosh(a+bx^n) \, dx$

3.53.1 Optimal result

Integrand size = 18, antiderivative size = 71

$$\int x^{-1+\frac{n}{2}} \cosh(a+bx^n) \, dx = \frac{e^{-a}\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}x^{n/2}\right)}{2\sqrt{b}n} + \frac{e^{a}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}x^{n/2}\right)}{2\sqrt{b}n}$$

output
```
1/2*erf(x^(1/2*n)*b^(1/2))*Pi^(1/2)/exp(a)/n/b^(1/2)+1/2*exp(a)*erfi(x^(1/
2*n)*b^(1/2))*Pi^(1/2)/n/b^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int x^{-1+\frac{n}{2}} \cosh(a+bx^n) \, dx = \frac{e^{-a}\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{b}x^{n/2}\right)+e^{2a}\operatorname{erfi}\left(\sqrt{b}x^{n/2}\right)\right)}{2\sqrt{b}n}$$

input
```
Integrate[x^(-1 + n/2)*Cosh[a + b*x^n],x]
```

output
```
(Sqrt[Pi]*(Erf[Sqrt[b]*x^(n/2)] + E^(2*a)*Erfi[Sqrt[b]*x^(n/2)]))/(2*Sqrt[
b]*E^a*n)
```

3.53.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5880, 5822, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{\frac{n}{2}-1}\cosh\left(a+bx^n\right)\,dx$$

$$\Big\downarrow \text{ 5880}$$

$$\frac{2\int \cosh\left(bx^n+a\right)dx^{n/2}}{n}$$

$$\Big\downarrow \text{ 5822}$$

$$\frac{2\left(\frac{1}{2}\int e^{-bx^n-a}dx^{n/2}+\frac{1}{2}\int e^{bx^n+a}dx^{n/2}\right)}{n}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{2\left(\frac{1}{2}\int e^{-bx^n-a}dx^{n/2}+\frac{\sqrt{\pi}e^a\text{erfi}\left(\sqrt{b}x^{n/2}\right)}{4\sqrt{b}}\right)}{n}$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{2\left(\frac{\sqrt{\pi}e^{-a}\text{erf}\left(\sqrt{b}x^{n/2}\right)}{4\sqrt{b}}+\frac{\sqrt{\pi}e^a\text{erfi}\left(\sqrt{b}x^{n/2}\right)}{4\sqrt{b}}\right)}{n}$$

input
```
Int[x^(-1 + n/2)*Cosh[a + b*x^n],x]
```

output
```
(2*((Sqrt[Pi]*Erf[Sqrt[b]*x^(n/2)])/(4*Sqrt[b]*E^a) + (E^a*Sqrt[Pi]*Erfi[S
qrt[b]*x^(n/2)])/(4*Sqrt[b])))/n
```

3.53.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 5822
```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[1/2   Int[E^(c + d*x^n
), x], x] + Simp[1/2   Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IG
tQ[n, 1]
```

rule 5880
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/(m + 1)   Subst[Int[(a + b*Cosh[c + d*x^Simplify[n/(m + 1)]])^
p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegerQ[p] &&
NeQ[m, -1] && IGtQ[Simplify[n/(m + 1)], 0] &&  !IntegerQ[n]
```

3.53.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

method	result
risch	$\frac{e^{-a}\sqrt{\pi}\,\operatorname{erf}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2n\sqrt{b}}+\frac{e^{a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b}\,x^{\frac{n}{2}}\right)}{2n\sqrt{-b}}$
meijerg	$\frac{\sqrt{2}\,\sqrt{\pi}\left(\frac{\sqrt{ib}\,\sqrt{2}\,\operatorname{erf}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2\sqrt{b}}+\frac{\sqrt{ib}\,\sqrt{2}\,\operatorname{erfi}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2\sqrt{b}}\right)\cosh(a)}{2\sqrt{ib}\,n}-\frac{i\sqrt{2}\,\sqrt{\pi}\left(-\frac{\sqrt{2}\,(ib)^{\frac{3}{2}}\operatorname{erf}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2b^{\frac{3}{2}}}+\frac{\sqrt{2}\,(ib)^{\frac{3}{2}}\operatorname{erfi}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2b^{\frac{3}{2}}}\right)\sinh(a)}{2\sqrt{ib}\,n}$

input
```
int(x^(-1+1/2*n)*cosh(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/2/n*exp(-a)*Pi^(1/2)/b^(1/2)*erf(x^(1/2*n)*b^(1/2))+1/2/n*exp(a)*Pi^(1/2
)/(-b)^(1/2)*erf((-b)^(1/2)*x^(1/2*n))
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.38

$$\int x^{-1+\frac{n}{2}}\cosh\left(a+bx^n\right)\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-b}(\cosh\left(a\right)+\sinh\left(a\right))\operatorname{erf}\left(\sqrt{-b}x\cosh\left(\frac{1}{2}\left(n-2\right)\log\left(x\right)\right)+\sqrt{-b}x\sinh\left(\frac{1}{2}\left(n-2\right)\log\left(x\right)\right)\right)-\sqrt{\pi}}{2\,bn}$$

input
```
integrate(x^(-1+1/2*n)*cosh(a+b*x^n),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)*x*cosh(1/2*(n - 2
)*log(x)) + sqrt(-b)*x*sinh(1/2*(n - 2)*log(x))) - sqrt(pi)*sqrt(b)*(cosh(
a) - sinh(a))*erf(sqrt(b)*x*cosh(1/2*(n - 2)*log(x)) + sqrt(b)*x*sinh(1/2*
(n - 2)*log(x))))/(b*n)
```

3.53.6 Sympy [F]

$$\int x^{-1+\frac{n}{2}}\cosh\left(a+bx^n\right)\,dx=\int x^{\frac{n}{2}-1}\cosh\left(a+bx^n\right)dx$$

input
```
integrate(x**(-1+1/2*n)*cosh(a+b*x**n),x)
```

output
```
Integral(x**(n/2 - 1)*cosh(a + b*x**n), x)
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int x^{-1+\frac{n}{2}}\cosh\left(a+bx^n\right)\,dx=\frac{\sqrt{\pi}x^{\frac{1}{2}n}\left(\operatorname{erf}\left(\sqrt{bx^n}\right)-1\right)e^{(-a)}}{2\sqrt{bx^n}n}+\frac{\sqrt{\pi}x^{\frac{1}{2}n}\left(\operatorname{erf}\left(\sqrt{-bx^n}\right)-1\right)e^{a}}{2\sqrt{-bx^n}n}$$

input
```
integrate(x^(-1+1/2*n)*cosh(a+b*x^n),x, algorithm="maxima")
```

output
```
1/2*sqrt(pi)*x^(1/2*n)*(erf(sqrt(b*x^n)) - 1)*e^(-a)/(sqrt(b*x^n)*n) + 1/2
*sqrt(pi)*x^(1/2*n)*(erf(sqrt(-b*x^n)) - 1)*e^a/(sqrt(-b*x^n)*n)
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int x^{-1+\frac{n}{2}}\cosh\left(a+bx^n\right)\,dx = -\frac{\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{b}\sqrt{x^n}\right)e^{(-a)}}{\sqrt{b}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-b}\sqrt{x^n}\right)e^{a}}{\sqrt{-b}}}{2\,n}$$

input
```
integrate(x^(-1+1/2*n)*cosh(a+b*x^n),x, algorithm="giac")
```

output
```
-1/2*(sqrt(pi)*erf(-sqrt(b)*sqrt(x^n))*e^(-a)/sqrt(b) + sqrt(pi)*erf(-sqrt
(-b)*sqrt(x^n))*e^a/sqrt(-b))/n
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1+\frac{n}{2}}\cosh\left(a+bx^n\right)\,dx = \int x^{\frac{n}{2}-1}\cosh\left(a+b\,x^n\right)\,dx$$

input
```
int(x^(n/2 - 1)*cosh(a + b*x^n),x)
```

output
```
int(x^(n/2 - 1)*cosh(a + b*x^n), x)
```

3.54 $\int x^2 \cosh\left((a+bx)^2\right)\,dx$

3.54.1 Optimal result

Integrand size = 12, antiderivative size = 113

$$\int x^2\cosh\left((a+bx)^2\right)\,dx=\frac{\sqrt{\pi}\,\mathrm{erf}(a+bx)}{8b^3}+\frac{a^2\sqrt{\pi}\,\mathrm{erf}(a+bx)}{4b^3}-\frac{\sqrt{\pi}\,\mathrm{erfi}(a+bx)}{8b^3}+\frac{a^2\sqrt{\pi}\,\mathrm{erfi}(a+bx)}{4b^3}-\frac{a\sinh\left((a+bx)^2\right)}{b^3}+\frac{(a+bx)\sinh\left((a+bx)^2\right)}{2b^3}$$

output
```
-a*sinh((b*x+a)^2)/b^3+1/2*(b*x+a)*sinh((b*x+a)^2)/b^3+1/8*erf(b*x+a)*Pi^(
1/2)/b^3+1/4*a^2*erf(b*x+a)*Pi^(1/2)/b^3-1/8*erfi(b*x+a)*Pi^(1/2)/b^3+1/4*
a^2*erfi(b*x+a)*Pi^(1/2)/b^3
```

3.54.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

$$\int x^2\cosh\left((a+bx)^2\right)\,dx=\frac{(1+2a^2)\sqrt{\pi}\,\mathrm{erf}(a+bx)+(-1+2a^2)\sqrt{\pi}\,\mathrm{erfi}(a+bx)-4(a-bx)\sinh\left((a+bx)^2\right)}{8b^3}$$

input
```
Integrate[x^2*Cosh[(a + b*x)^2],x]
```

output
```
((1 + 2*a^2)*Sqrt[Pi]*Erf[a + b*x] + (-1 + 2*a^2)*Sqrt[Pi]*Erfi[a + b*x] -
 4*(a - b*x)*Sinh[(a + b*x)^2])/(8*b^3)
```

3.54.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5888, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cosh\left((a+bx)^2\right) dx$$

$$\Big\downarrow \; 5888$$

$$\frac{\int b^2x^2\cosh\left((a+bx)^2\right)d(a+bx)}{b^3}$$

$$\Big\downarrow \; 7293$$

$$\frac{\int \left(\cosh\left((a+bx)^2\right)a^2-2(a+bx)\cosh\left((a+bx)^2\right)a+(a+bx)^2\cosh\left((a+bx)^2\right)\right)d(a+bx)}{b^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{4}\sqrt{\pi}a^2\text{erf}(a+bx)+\frac{1}{4}\sqrt{\pi}a^2\text{erfi}(a+bx)+\frac{1}{8}\sqrt{\pi}\text{erf}(a+bx)-\frac{1}{8}\sqrt{\pi}\text{erfi}(a+bx)-a\sinh\left((a+bx)^2\right)+\frac{1}{2}(a+bx)\,\text{sin}}{b^3}$$

input
```
Int[x^2*Cosh[(a + b*x)^2],x]
```

output
```
((Sqrt[Pi]*Erf[a + b*x])/8 + (a^2*Sqrt[Pi]*Erf[a + b*x])/4 - (Sqrt[Pi]*Erf
i[a + b*x])/8 + (a^2*Sqrt[Pi]*Erfi[a + b*x])/4 - a*Sinh[(a + b*x)^2] + ((a
 + b*x)*Sinh[(a + b*x)^2])/2)/b^3
```

3.54.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5888
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/Coefficient[u, x, 1]^(m + 1)   Subst[Int[(x - Coefficient[u, x
, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p
}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.54.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{x\,\mathrm{e}^{-(bx+a)^2}}{4b^2}+\frac{a\,\mathrm{e}^{-(bx+a)^2}}{4b^3}+\frac{a^2\,\mathrm{erf}(bx+a)\sqrt{\pi}}{4b^3}+\frac{\mathrm{erf}(bx+a)\sqrt{\pi}}{8b^3}+\frac{x\,\mathrm{e}^{(bx+a)^2}}{4b^2}-\frac{a\,\mathrm{e}^{(bx+a)^2}}{4b^3}-\frac{ia^2\sqrt{\pi}\,\mathrm{erf}(ixb+ia)}{4b^3}+ i\sqrt{}$

input
```
int(x^2*cosh((b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^2*x*exp(-(b*x+a)^2)+1/4*a/b^3*exp(-(b*x+a)^2)+1/4*a^2*erf(b*x+a)*Pi
^(1/2)/b^3+1/8*erf(b*x+a)*Pi^(1/2)/b^3+1/4/b^2*x*exp((b*x+a)^2)-1/4*a/b^3*
exp((b*x+a)^2)-1/4*I*a^2/b^3*Pi^(1/2)*erf(I*x*b+I*a)+1/8*I/b^3*Pi^(1/2)*er
f(I*x*b+I*a)
```

3.54.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(95) = 190$.

Time = 0.26 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.84

$$\int x^2 \cosh\left((a+bx)^2\right) dx =$$

$$-\frac{2\,b^2x-2\,(b^2x-ab)\cosh\left(b^2x^2+2\,abx+a^2\right)^2+\sqrt{\pi}\sqrt{-b^2}\left((2\,a^2-1)\cosh\left(b^2x^2+2\,abx+a^2\right)+(2\,a^2-\right.}{}$$

input
```
integrate(x^2*cosh((b*x+a)^2),x, algorithm="fricas")
```

output
```
-1/8*(2*b^2*x - 2*(b^2*x - a*b)*cosh(b^2*x^2 + 2*a*b*x + a^2)^2 + sqrt(pi)
*sqrt(-b^2)*((2*a^2 - 1)*cosh(b^2*x^2 + 2*a*b*x + a^2) + (2*a^2 - 1)*sinh(
b^2*x^2 + 2*a*b*x + a^2))*erf(sqrt(-b^2)*(b*x + a)/b) - sqrt(pi)*sqrt(b^2)
*((2*a^2 + 1)*cosh(b^2*x^2 + 2*a*b*x + a^2) + (2*a^2 + 1)*sinh(b^2*x^2 + 2
*a*b*x + a^2))*erf(sqrt(b^2)*(b*x + a)/b) - 4*(b^2*x - a*b)*cosh(b^2*x^2 +
 2*a*b*x + a^2)*sinh(b^2*x^2 + 2*a*b*x + a^2) - 2*(b^2*x - a*b)*sinh(b^2*x
^2 + 2*a*b*x + a^2)^2 - 2*a*b)/(b^4*cosh(b^2*x^2 + 2*a*b*x + a^2) + b^4*si
nh(b^2*x^2 + 2*a*b*x + a^2))
```

3.54.6 Sympy [F]

$$\int x^2 \cosh\left((a+bx)^2\right)\,dx = \int x^2 \cosh\left(a^2+2abx+b^2x^2\right)dx$$

input
```
integrate(x**2*cosh((b*x+a)**2),x)
```

output
```
Integral(x**2*cosh(a**2 + 2*a*b*x + b**2*x**2), x)
```

3.54.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(95) = 190$.

Time = 0.44 (sec) , antiderivative size = 818, normalized size of antiderivative = 7.24

$$\int x^2 \cosh\left((a+bx)^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*cosh((b*x+a)^2),x, algorithm="maxima")
```

output
```
1/3*x^3*cosh((b*x + a)^2) - 1/6*((sqrt(pi)*(b^2*x + a*b)*a^3*b^4*(erf(sqrt
((b^2*x + a*b)^2)/b) - 1)/(sqrt((b^2*x + a*b)^2)*(-b^2)^(7/2)) - 3*(b^2*x
+ a*b)^3*a*b^4*gamma(3/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a*b)^2)^(3/2)*(-
b^2)^(7/2)) + 3*a^2*b^4*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(7/2) + b^4*gamma(
2, (b^2*x + a*b)^2/b^2)/(-b^2)^(7/2))*a/sqrt(-b^2) + (sqrt(pi)*(b^2*x + a*
b)*a^4*b^5*(erf(sqrt((b^2*x + a*b)^2)/b) - 1)/(sqrt((b^2*x + a*b)^2)*(-b^2
)^(9/2)) - 6*(b^2*x + a*b)^3*a^2*b^5*gamma(3/2, (b^2*x + a*b)^2/b^2)/(((b^
2*x + a*b)^2)^(3/2)*(-b^2)^(9/2)) + 4*a^3*b^5*e^(-(b^2*x + a*b)^2/b^2)/(-b
^2)^(9/2) - (b^2*x + a*b)^5*b^5*gamma(5/2, (b^2*x + a*b)^2/b^2)/(((b^2*x +
 a*b)^2)^(5/2)*(-b^2)^(9/2)) + 4*a*b^5*gamma(2, (b^2*x + a*b)^2/b^2)/(-b^2
)^(9/2))*b/sqrt(-b^2) - a*(sqrt(pi)*(b^2*x + a*b)*a^3*(erf(sqrt(-(b^2*x +
a*b)^2/b^2)) - 1)/(b^4*sqrt(-(b^2*x + a*b)^2/b^2)) - 3*a^2*e^((b^2*x + a*b
)^2/b^2)/b^3 + gamma(2, -(b^2*x + a*b)^2/b^2)/b^3 - 3*(b^2*x + a*b)^3*a*ga
mma(3/2, -(b^2*x + a*b)^2/b^2)/(b^6*(-(b^2*x + a*b)^2/b^2)^(3/2)))/b + sqr
t(pi)*(b^2*x + a*b)*a^4*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) - 1)/(b^5*sqrt(-(
b^2*x + a*b)^2/b^2)) - 4*a^3*e^((b^2*x + a*b)^2/b^2)/b^4 + 4*a*gamma(2, -(
b^2*x + a*b)^2/b^2)/b^4 - 6*(b^2*x + a*b)^3*a^2*gamma(3/2, -(b^2*x + a*b)^
2/b^2)/(b^7*(-(b^2*x + a*b)^2/b^2)^(3/2)) - (b^2*x + a*b)^5*gamma(5/2, -(b
^2*x + a*b)^2/b^2)/(b^9*(-(b^2*x + a*b)^2/b^2)^(5/2)))*b
```

3.54.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21

$$\int x^2 \cosh \left((a+bx)^2\right) \, dx = -\frac{\frac{i\sqrt{\pi}\left(2a^2-1\right)\operatorname{erf}\left(ib\left(x+\frac{a}{b}\right)\right)}{b} - \frac{2\left(b\left(x+\frac{a}{b}\right)-2a\right)e^{\left(b^2x^2+2abx+a^2\right)}}{b}}{8b^2} - \frac{\frac{\sqrt{\pi}\left(2a^2+1\right)\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} + \frac{2\left(b\left(x+\frac{a}{b}\right)-2a\right)e^{\left(-b^2x^2-2abx-a^2\right)}}{b}}{8b^2}$$

input
```
integrate(x^2*cosh((b*x+a)^2),x, algorithm="giac")
```

output
```
-1/8*(I*sqrt(pi)*(2*a^2 - 1)*erf(I*b*(x + a/b))/b - 2*(b*(x + a/b) - 2*a)*
e^(b^2*x^2 + 2*a*b*x + a^2)/b)/b^2 - 1/8*(sqrt(pi)*(2*a^2 + 1)*erf(-b*(x +
 a/b))/b + 2*(b*(x + a/b) - 2*a)*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/b^2
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh \left((a+bx)^2\right) \, dx = \int x^2 \cosh\left((a+b\,x)^2\right) \, dx$$

input
```
int(x^2*cosh((a + b*x)^2),x)
```

output
```
int(x^2*cosh((a + b*x)^2), x)
```

3.55 $\int x \cosh \left((a+bx)^2\right) dx$

3.55.1 Optimal result

Integrand size = 10, antiderivative size = 54

$$\int x \cosh \left((a+bx)^2\right) dx = -\frac{a\sqrt{\pi}\text{erf}(a+bx)}{4b^2} - \frac{a\sqrt{\pi}\text{erfi}(a+bx)}{4b^2} + \frac{\sinh\left((a+bx)^2\right)}{2b^2}$$

output
```
1/2*sinh((b*x+a)^2)/b^2-1/4*a*erf(b*x+a)*Pi^(1/2)/b^2-1/4*a*erfi(b*x+a)*Pi
^(1/2)/b^2
```

3.55.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.72

$$\int x \cosh \left((a+bx)^2\right) dx = \frac{-a\sqrt{\pi}(\text{erf}(a+bx)+\text{erfi}(a+bx)) + 2\sinh\left((a+bx)^2\right)}{4b^2}$$

input
```
Integrate[x*Cosh[(a + b*x)^2],x]
```

output
```
(-(a*Sqrt[Pi]*(Erf[a + b*x] + Erfi[a + b*x])) + 2*Sinh[(a + b*x)^2])/(4*b^
2)
```

3.55.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5888, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh\left((a+bx)^2\right)\,dx$$

$$\Big\downarrow \text{ 5888}$$

$$\frac{\int bx\cosh\left((a+bx)^2\right)d(a+bx)}{b^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -bx\cosh\left((a+bx)^2\right)d(a+bx)}{b^2}$$

$$\Big\downarrow \text{ 7293}$$

$$-\frac{\int \left(a\cosh\left((a+bx)^2\right)-(a+bx)\cosh\left((a+bx)^2\right)\right)d(a+bx)}{b^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{1}{4}\sqrt{\pi}a\text{erf}(a+bx)-\frac{1}{4}\sqrt{\pi}a\text{erfi}(a+bx)+\frac{1}{2}\sinh\left((a+bx)^2\right)}{b^2}$$

input
```
Int[x*Cosh[(a + b*x)^2],x]
```

output
```
(-1/4*(a*Sqrt[Pi]*Erf[a + b*x]) - (a*Sqrt[Pi]*Erfi[a + b*x])/4 + Sinh[(a +
 b*x)^2]/2)/b^2
```

3.55.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5888
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/Coefficient[u, x, 1]^(m + 1)   Subst[Int[(x - Coefficient[u, x
, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p
}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.55.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

method	result	size
risch	$-\frac{e^{-(bx+a)^2}}{4b^2}-\frac{a\,\mathrm{erf}(bx+a)\sqrt{\pi}}{4b^2}+\frac{e^{(bx+a)^2}}{4b^2}+\frac{ia\sqrt{\pi}\,\mathrm{erf}(ixb+ia)}{4b^2}$	66

input
```
int(x*cosh((b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^2*exp(-(b*x+a)^2)-1/4*a*erf(b*x+a)*Pi^(1/2)/b^2+1/4/b^2*exp((b*x+a)
^2)+1/4*I*a/b^2*Pi^(1/2)*erf(I*x*b+I*a)
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(44) = 88.

Time = 0.26 (sec) , antiderivative size = 261, normalized size of antiderivative = 4.83

$$\int x\cosh\left((a+bx)^2\right)\,dx$$
$$=\frac{b\cosh\left(b^2x^2+2\,abx+a^2\right)^2+\sqrt{\pi}\sqrt{-b^2}\left(a\cosh\left(b^2x^2+2\,abx+a^2\right)+a\sinh\left(b^2x^2+2\,abx+a^2\right)\right)\mathrm{erf}\left(\frac{\sqrt{-b^2}(}{b}\right.}{}$$

input
```
integrate(x*cosh((b*x+a)^2),x, algorithm="fricas")
```

output
```
1/4*(b*cosh(b^2*x^2 + 2*a*b*x + a^2)^2 + sqrt(pi)*sqrt(-b^2)*(a*cosh(b^2*x
^2 + 2*a*b*x + a^2) + a*sinh(b^2*x^2 + 2*a*b*x + a^2))*erf(sqrt(-b^2)*(b*x
 + a)/b) - sqrt(pi)*sqrt(b^2)*(a*cosh(b^2*x^2 + 2*a*b*x + a^2) + a*sinh(b^
2*x^2 + 2*a*b*x + a^2))*erf(sqrt(b^2)*(b*x + a)/b) + 2*b*cosh(b^2*x^2 + 2*
a*b*x + a^2)*sinh(b^2*x^2 + 2*a*b*x + a^2) + b*sinh(b^2*x^2 + 2*a*b*x + a^
2)^2 - b)/(b^3*cosh(b^2*x^2 + 2*a*b*x + a^2) + b^3*sinh(b^2*x^2 + 2*a*b*x
+ a^2))
```

3.55.6 Sympy [F]

$$\int x\cosh\left((a+bx)^2\right)\,dx=\int x\cosh\left(a^2+2abx+b^2x^2\right)dx$$

input
```
integrate(x*cosh((b*x+a)**2),x)
```

output
```
Integral(x*cosh(a**2 + 2*a*b*x + b**2*x**2), x)
```

3.55.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. 2(44) = 88.

Time = 0.40 (sec) , antiderivative size = 650, normalized size of antiderivative = 12.04

$$\int x\cosh\left((a+bx)^2\right)\,dx=\frac{1}{2}x^2\cosh\left((bx+a)^2\right)-\frac{1}{4}\left(\frac{\left(\frac{\sqrt{\pi}(b^2x+ab)a^2b^3\left(\operatorname{erf}\left(\frac{\sqrt{(b^2x+ab)^2}}{b}\right)-1\right)}{\sqrt{(b^2x+ab)^2}(-b^2)^{\frac{5}{2}}}-\frac{(b^2x+ab)^3b^3\Gamma\left(\frac{3}{2},\frac{(b^2x+ab)^2}{b^2}\right)}{\left((b^2x+ab)^2\right)^{\frac{3}{2}}(-b^2)^{\frac{5}{2}}}+\frac{2ab^3e^{\left(-\frac{(b^2x+ab)^2}{b^2}\right)}}{(-b^2)^{\frac{5}{2}}}\right)a}{\sqrt{-b^2}}+\frac{\left(\frac{\sqrt{\pi}(b^2x+ab}{}\right.}{}\right.$$

input
```
integrate(x*cosh((b*x+a)^2),x, algorithm="maxima")
```

output
```
1/2*x^2*cosh((b*x + a)^2) - 1/4*((sqrt(pi)*(b^2*x + a*b)*a^2*b^3*(erf(sqrt
((b^2*x + a*b)^2)/b) - 1)/(sqrt((b^2*x + a*b)^2)*(-b^2)^(5/2)) - (b^2*x +
a*b)^3*b^3*gamma(3/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a*b)^2)^(3/2)*(-b^2)
^(5/2)) + 2*a*b^3*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(5/2))*a/sqrt(-b^2) + (s
qrt(pi)*(b^2*x + a*b)*a^3*b^4*(erf(sqrt((b^2*x + a*b)^2)/b) - 1)/(sqrt((b^
2*x + a*b)^2)*(-b^2)^(7/2)) - 3*(b^2*x + a*b)^3*a*b^4*gamma(3/2, (b^2*x +
a*b)^2/b^2)/(((b^2*x + a*b)^2)^(3/2)*(-b^2)^(7/2)) + 3*a^2*b^4*e^(-(b^2*x
+ a*b)^2/b^2)/(-b^2)^(7/2) + b^4*gamma(2, (b^2*x + a*b)^2/b^2)/(-b^2)^(7/2
))*b/sqrt(-b^2) + a*(sqrt(pi)*(b^2*x + a*b)*a^2*(erf(sqrt(-(b^2*x + a*b)^2
/b^2)) - 1)/(b^3*sqrt(-(b^2*x + a*b)^2/b^2)) - 2*a*e^((b^2*x + a*b)^2/b^2)
/b^2 - (b^2*x + a*b)^3*gamma(3/2, -(b^2*x + a*b)^2/b^2)/(b^5*(-(b^2*x + a*
b)^2/b^2)^(3/2)))/b - sqrt(pi)*(b^2*x + a*b)*a^3*(erf(sqrt(-(b^2*x + a*b)^
2/b^2)) - 1)/(b^4*sqrt(-(b^2*x + a*b)^2/b^2)) + 3*a^2*e^((b^2*x + a*b)^2/b
^2)/b^3 - gamma(2, -(b^2*x + a*b)^2/b^2)/b^3 + 3*(b^2*x + a*b)^3*a*gamma(3
/2, -(b^2*x + a*b)^2/b^2)/(b^6*(-(b^2*x + a*b)^2/b^2)^(3/2)))*b
```

3.55.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.83

$$\int x\cosh\left((a+bx)^2\right)\,dx = -\frac{-\frac{i\sqrt{\pi}a\operatorname{erf}\left(ib\left(x+\frac{a}{b}\right)\right)}{b}-\frac{e^{\left(b^2x^2+2\,abx+a^2\right)}}{b}}{4\,b} + \frac{\frac{\sqrt{\pi}a\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b}-\frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{4\,b}$$

input
```
integrate(x*cosh((b*x+a)^2),x, algorithm="giac")
```

output
```
-1/4*(-I*sqrt(pi)*a*erf(I*b*(x + a/b))/b - e^(b^2*x^2 + 2*a*b*x + a^2)/b)/
b + 1/4*(sqrt(pi)*a*erf(-b*(x + a/b))/b - e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/
b
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int x \cosh \left((a+bx)^2\right) \, dx = \int x \cosh\left((a+b\,x)^2\right) \, dx$$

input
```
int(x*cosh((a + b*x)^2),x)
```

output
```
int(x*cosh((a + b*x)^2), x)
```

3.56 $\int \cosh\left((a+bx)^2\right)\,dx$

3.56.1 Optimal result

Integrand size = 8, antiderivative size = 37

$$\int \cosh\left((a+bx)^2\right)\,dx=\frac{\sqrt{\pi}\,\mathrm{erf}(a+bx)}{4b}+\frac{\sqrt{\pi}\,\mathrm{erfi}(a+bx)}{4b}$$

output
```
1/4*erf(b*x+a)*Pi^(1/2)/b+1/4*erfi(b*x+a)*Pi^(1/2)/b
```

3.56.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \cosh\left((a+bx)^2\right)\,dx=\frac{\sqrt{\pi}(\mathrm{erf}(a+bx)+\mathrm{erfi}(a+bx))}{4b}$$

input
```
Integrate[Cosh[(a + b*x)^2],x]
```

output
```
(Sqrt[Pi]*(Erf[a + b*x] + Erfi[a + b*x]))/(4*b)
```

3.56.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5834, 5822, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh\left((a+bx)^2\right) dx$$

$$\Big\downarrow \text{5834}$$

$$\frac{\int \cosh\left((a+bx)^2\right) d(a+bx)}{b}$$

$$\Big\downarrow \text{5822}$$

$$\frac{\frac{1}{2}\int e^{-(a+bx)^2}d(a+bx)+\frac{1}{2}\int e^{(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\frac{1}{2}\int e^{-(a+bx)^2}d(a+bx)+\frac{1}{4}\sqrt{\pi}\text{erfi}(a+bx)}{b}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\frac{1}{4}\sqrt{\pi}\text{erf}(a+bx)+\frac{1}{4}\sqrt{\pi}\text{erfi}(a+bx)}{b}$$

input
```
Int[Cosh[(a + b*x)^2],x]
```

output
```
((Sqrt[Pi]*Erf[a + b*x])/4 + (Sqrt[Pi]*Erfi[a + b*x])/4)/b
```

3.56.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 5822
```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[1/2   Int[E^(c + d*x^n
), x], x] + Simp[1/2   Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IG
tQ[n, 1]
```

rule 5834
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.), x_Symbol] :> Simp[
1/Coefficient[u, x, 1]   Subst[Int[(a + b*Cosh[c + d*x^n])^p, x], x, u], x]
 /; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

3.56.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{\operatorname{erf}(bx+a)\sqrt{\pi}}{4b}-\frac{i\sqrt{\pi}\,\operatorname{erf}(ixb+ia)}{4b}$	36

input
```
int(cosh((b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*erf(b*x+a)*Pi^(1/2)/b-1/4*I*Pi^(1/2)/b*erf(I*x*b+I*a)
```

3.56.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(29) = 58.

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int \cosh\left((a+bx)^2\right)\,dx=-\frac{\sqrt{\pi}\sqrt{-b^2}\operatorname{erf}\left(\frac{\sqrt{-b^2}(bx+a)}{b}\right)-\sqrt{\pi}\sqrt{b^2}\operatorname{erf}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)}{4\,b^2}$$

input
```
integrate(cosh((b*x+a)^2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*sqrt(-b^2)*erf(sqrt(-b^2)*(b*x + a)/b) - sqrt(pi)*sqrt(b^2)
*erf(sqrt(b^2)*(b*x + a)/b))/b^2
```

3.56.6 Sympy [F]

$$\int \cosh\left((a+bx)^2\right)\,dx = \int \cosh\left((a+bx)^2\right)dx$$

input
```
integrate(cosh((b*x+a)**2),x)
```

output
```
Integral(cosh((a + b*x)**2), x)
```

3.56.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(29) = 58$.

Time = 0.39 (sec) , antiderivative size = 478, normalized size of antiderivative = 12.92

$$\int \cosh\left((a+bx)^2\right)\,dx = -\frac{1}{2}\left(\frac{\left(\frac{\sqrt{\pi}(b^2x+ab)ab^2\left(\operatorname{erf}\left(\frac{\sqrt{(b^2x+ab)^2}}{b}\right)-1\right)}{\sqrt{(b^2x+ab)^2}(-b^2)^{\frac{3}{2}}}+\frac{b^2e^{\left(-\frac{(b^2x+ab)^2}{b^2}\right)}}{(-b^2)^{\frac{3}{2}}}\right)a}{\sqrt{-b^2}}+\frac{\left(\frac{\sqrt{\pi}(b^2x+ab)a^2b^3\left(\operatorname{erf}\left(\frac{\sqrt{(b^2x+ab)^2}}{b}\right)-1\right)}{\sqrt{(b^2x+ab)^2}(-b^2)^{\frac{5}{2}}}-\frac{(b^2}{}\right.}{\sqrt{}}\right. + x\cosh\left((bx+a)^2\right)$$

input
```
integrate(cosh((b*x+a)^2),x, algorithm="maxima")
```

output
```
-1/2*((sqrt(pi)*(b^2*x + a*b)*a*b^2*(erf(sqrt((b^2*x + a*b)^2)/b) - 1)/(sq
rt((b^2*x + a*b)^2)*(-b^2)^(3/2)) + b^2*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(3
/2))*a/sqrt(-b^2) + (sqrt(pi)*(b^2*x + a*b)*a^2*b^3*(erf(sqrt((b^2*x + a*b
)^2)/b) - 1)/(sqrt((b^2*x + a*b)^2)*(-b^2)^(5/2)) - (b^2*x + a*b)^3*b^3*ga
mma(3/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a*b)^2)^(3/2)*(-b^2)^(5/2)) + 2*a
*b^3*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(5/2))*b/sqrt(-b^2) - a*(sqrt(pi)*(b^
2*x + a*b)*a*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) - 1)/(b^2*sqrt(-(b^2*x + a*b
)^2/b^2)) - e^((b^2*x + a*b)^2/b^2)/b)/b + sqrt(pi)*(b^2*x + a*b)*a^2*(erf
(sqrt(-(b^2*x + a*b)^2/b^2)) - 1)/(b^3*sqrt(-(b^2*x + a*b)^2/b^2)) - 2*a*e
^((b^2*x + a*b)^2/b^2)/b^2 - (b^2*x + a*b)^3*gamma(3/2, -(b^2*x + a*b)^2/b
^2)/(b^5*(-(b^2*x + a*b)^2/b^2)^(3/2)))*b + x*cosh((b*x + a)^2)
```

3.56.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \cosh\left((a+bx)^2\right)\,dx = -\frac{i\sqrt{\pi}\,\mathrm{erf}\left(i\,b\left(x+\frac{a}{b}\right)\right)}{4\,b} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{4\,b}$$

input
```
integrate(cosh((b*x+a)^2),x, algorithm="giac")
```

output
```
-1/4*I*sqrt(pi)*erf(I*b*(x + a/b))/b - 1/4*sqrt(pi)*erf(-b*(x + a/b))/b
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \cosh\left((a+bx)^2\right)\,dx = \int \cosh\left({\left(a+b\,x\right)}^2\right)\,dx$$

input
```
int(cosh((a + b*x)^2),x)
```

output
```
int(cosh((a + b*x)^2), x)
```

3.57 $\int \frac{\cosh\left((a+bx)^2\right)}{x} \, dx$

3.57.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\cosh\left((a+bx)^2\right)}{x} \, dx = b\text{Int}\left(\frac{\cosh\left((a+bx)^2\right)}{bx}, x\right)$$

output
```
b*CannotIntegrate(cosh((b*x+a)^2)/b/x,x)
```

3.57.2 Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh\left((a+bx)^2\right)}{x} \, dx = \int \frac{\cosh\left((a+bx)^2\right)}{x} \, dx$$

input
```
Integrate[Cosh[(a + b*x)^2]/x,x]
```

output
```
Integrate[Cosh[(a + b*x)^2]/x, x]
```

3.57.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5888, 25, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh \left((a+bx)^2\right)}{x} \, dx$$

$$\Big\downarrow \text{5888}$$

$$\int \frac{\cosh \left((a+bx)^2\right)}{bx} d(a+bx)$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{\cosh \left((a+bx)^2\right)}{bx} d(a+bx)$$

$$\Big\downarrow \text{7299}$$

$$-\int -\frac{\cosh \left((a+bx)^2\right)}{bx} d(a+bx)$$

input
```
Int[Cosh[(a + b*x)^2]/x,x]
```

output
```
$Aborted
```

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 5888
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/Coefficient[u, x, 1]^(m + 1)   Subst[Int[(x - Coefficient[u, x
, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p
}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.57.4 Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left((bx+a)^2\right)}{x}dx$$

input
```
int(cosh((b*x+a)^2)/x,x)
```

output
```
int(cosh((b*x+a)^2)/x,x)
```

3.57.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{\cosh((a+bx)^2)}{x}\,dx = \int \frac{\cosh\left((bx+a)^2\right)}{x}\,dx$$

input
```
integrate(cosh((b*x+a)^2)/x,x, algorithm="fricas")
```

output
```
integral(cosh(b^2*x^2 + 2*a*b*x + a^2)/x, x)
```

3.57.6 Sympy [N/A]

Not integrable

Time = 3.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\cosh((a+bx)^2)}{x}\,dx = \int \frac{\cosh(a^2+2abx+b^2x^2)}{x}\,dx$$

input
```
integrate(cosh((b*x+a)**2)/x,x)
```

output
```
Integral(cosh(a**2 + 2*a*b*x + b**2*x**2)/x, x)
```

3.57.7 Maxima [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh ((a+bx)^2)}{x} dx = \int \frac{\cosh \left((bx+a)^2\right)}{x} dx$$

input
```
integrate(cosh((b*x+a)^2)/x,x, algorithm="maxima")
```

output
```
integrate(cosh((b*x + a)^2)/x, x)
```

3.57.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh ((a+bx)^2)}{x} dx = \int \frac{\cosh \left((bx+a)^2\right)}{x} dx$$

input
```
integrate(cosh((b*x+a)^2)/x,x, algorithm="giac")
```

output
```
integrate(cosh((b*x + a)^2)/x, x)
```

3.57.9 Mupad [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh ((a+bx)^2)}{x} dx = \int \frac{\cosh\left((a+b\,x)^2\right)}{x} dx$$

input
```
int(cosh((a + b*x)^2)/x,x)
```

output
```
int(cosh((a + b*x)^2)/x, x)
```

3.58 $\int \frac{\cosh\left((a+bx)^2\right)}{x^2}\,dx$

3.58.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\cosh\left((a+bx)^2\right)}{x^2}\,dx = \operatorname{Int}\left(\frac{\cosh\left((a+bx)^2\right)}{x^2},x\right)$$

output
```
Unintegrable(cosh((b*x+a)^2)/x^2,x)
```

3.58.2 Mathematica [N/A]

Not integrable

Time = 3.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh\left((a+bx)^2\right)}{x^2}\,dx = \int \frac{\cosh\left((a+bx)^2\right)}{x^2}\,dx$$

input
```
Integrate[Cosh[(a + b*x)^2]/x^2,x]
```

output
```
Integrate[Cosh[(a + b*x)^2]/x^2, x]
```

3.58.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5888, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh\left((a+bx)^2\right)}{x^2}dx$$

$$\Big\downarrow \text{ 5888}$$

$$b\int \frac{\cosh\left((a+bx)^2\right)}{b^2x^2}d(a+bx)$$

$$\Big\downarrow \text{ 7299}$$

$$b\int \frac{\cosh\left((a+bx)^2\right)}{b^2x^2}d(a+bx)$$

input

```
Int[Cosh[(a + b*x)^2]/x^2,x]
```

output

```
$Aborted
```

3.58.3.1 Defintions of rubi rules used

rule 5888

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/Coefficient[u, x, 1]^(m + 1)   Subst[Int[(x - Coefficient[u, x
, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p
}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

rule 7299

```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.58.4 Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left((bx+a)^2\right)}{x^2}dx$$

input
```
int(cosh((b*x+a)^2)/x^2,x)
```

output
```
int(cosh((b*x+a)^2)/x^2,x)
```

3.58.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{\cosh\left((a+bx)^2\right)}{x^2}\,dx = \int \frac{\cosh\left((bx+a)^2\right)}{x^2}\,dx$$

input
```
integrate(cosh((b*x+a)^2)/x^2,x, algorithm="fricas")
```

output
```
integral(cosh(b^2*x^2 + 2*a*b*x + a^2)/x^2, x)
```

3.58.6 Sympy [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\cosh\left((a+bx)^2\right)}{x^2}\,dx = \int \frac{\cosh\left(a^2+2abx+b^2x^2\right)}{x^2}\,dx$$

input
```
integrate(cosh((b*x+a)**2)/x**2,x)
```

output
```
Integral(cosh(a**2 + 2*a*b*x + b**2*x**2)/x**2, x)
```

3.58.7 Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh \left((a+bx)^2\right)}{x^2} \, dx = \int \frac{\cosh \left((bx+a)^2\right)}{x^2} \, dx$$

input
```
integrate(cosh((b*x+a)^2)/x^2,x, algorithm="maxima")
```

output
```
integrate(cosh((b*x + a)^2)/x^2, x)
```

3.58.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh \left((a+bx)^2\right)}{x^2} \, dx = \int \frac{\cosh \left((bx+a)^2\right)}{x^2} \, dx$$

input
```
integrate(cosh((b*x+a)^2)/x^2,x, algorithm="giac")
```

output
```
integrate(cosh((b*x + a)^2)/x^2, x)
```

3.58.9 Mupad [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh \left((a+bx)^2\right)}{x^2} \, dx = \int \frac{\cosh \left({\left(a+b\,x\right)}^2\right)}{x^2} \, dx$$

input
```
int(cosh((a + b*x)^2)/x^2,x)
```

output
```
int(cosh((a + b*x)^2)/x^2, x)
```

3.59 $\int x^2 \cosh\left(a+b\sqrt{c+dx}\right)\,dx$

3.59.1 Optimal result

Integrand size = 18, antiderivative size = 346

$$\begin{aligned}\int x^2\cosh\left(a+b\sqrt{c+dx}\right)dx =& -\frac{240\cosh\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{24c\cosh\left(a+b\sqrt{c+dx}\right)}{b^4d^3}\\ &-\frac{2c^2\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}\\ &-\frac{120(c+dx)\cosh\left(a+b\sqrt{c+dx}\right)}{b^4d^3}\\ &+\frac{12c(c+dx)\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}\\ &-\frac{10(c+dx)^2\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}\\ &+\frac{240\sqrt{c+dx}\sinh\left(a+b\sqrt{c+dx}\right)}{b^5d^3}\\ &-\frac{24c\sqrt{c+dx}\sinh\left(a+b\sqrt{c+dx}\right)}{b^3d^3}\\ &+\frac{2c^2\sqrt{c+dx}\sinh\left(a+b\sqrt{c+dx}\right)}{bd^3}\\ &+\frac{40(c+dx)^{3/2}\sinh\left(a+b\sqrt{c+dx}\right)}{b^3d^3}\\ &-\frac{4c(c+dx)^{3/2}\sinh\left(a+b\sqrt{c+dx}\right)}{bd^3}\\ &+\frac{2(c+dx)^{5/2}\sinh\left(a+b\sqrt{c+dx}\right)}{bd^3}\end{aligned}$$

output
```
-240*cosh(a+b*(d*x+c)^(1/2))/b^6/d^3+24*c*cosh(a+b*(d*x+c)^(1/2))/b^4/d^3-
2*c^2*cosh(a+b*(d*x+c)^(1/2))/b^2/d^3-120*(d*x+c)*cosh(a+b*(d*x+c)^(1/2))/
b^4/d^3+12*c*(d*x+c)*cosh(a+b*(d*x+c)^(1/2))/b^2/d^3-10*(d*x+c)^2*cosh(a+b
*(d*x+c)^(1/2))/b^2/d^3+40*(d*x+c)^(3/2)*sinh(a+b*(d*x+c)^(1/2))/b^3/d^3-4
*c*(d*x+c)^(3/2)*sinh(a+b*(d*x+c)^(1/2))/b/d^3+2*(d*x+c)^(5/2)*sinh(a+b*(d
*x+c)^(1/2))/b/d^3+240*sinh(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^5/d^3-24*c*
sinh(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^3/d^3+2*c^2*sinh(a+b*(d*x+c)^(1/2)
)*(d*x+c)^(1/2)/b/d^3
```

3.59.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.60

$$\int x^2\cosh\left(a+b\sqrt{c+dx}\right)\,dx$$
$$=\frac{e^{a+b\sqrt{c+dx}}\left(-120+120b\sqrt{c+dx}+b^5d^2x^2\sqrt{c+dx}+4b^3\sqrt{c+dx}(2c+5dx)-12b^2(4c+5dx)-b^4dx(4c+\cdots\right.}{\cdots}$$

input
```
Integrate[x^2*Cosh[a + b*Sqrt[c + d*x]],x]
```

output
```
(E^(a + b*Sqrt[c + d*x])*(-120 + 120*b*Sqrt[c + d*x] + b^5*d^2*x^2*Sqrt[c
+ d*x] + 4*b^3*Sqrt[c + d*x]*(2*c + 5*d*x) - 12*b^2*(4*c + 5*d*x) - b^4*d*
x*(4*c + 5*d*x)) - E^(-a - b*Sqrt[c + d*x])*(120 + 120*b*Sqrt[c + d*x] + b
^5*d^2*x^2*Sqrt[c + d*x] + 4*b^3*Sqrt[c + d*x]*(2*c + 5*d*x) + 12*b^2*(4*c
 + 5*d*x) + b^4*d*x*(4*c + 5*d*x)))/(b^6*d^3)
```

3.59.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5888, 7267, 5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\cosh\left(a+b\sqrt{c+dx}\right)\,dx$$

$$\Big\downarrow \text{5888}$$

$$\frac{\int d^2x^2\cosh\left(a+b\sqrt{c+dx}\right)d(c+dx)}{d^3}$$

$$\Big\downarrow \ 7267$$

$$\frac{2\int d^2x^2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)d\sqrt{c+dx}}{d^3}$$

$$\Big\downarrow \ 5810$$

$$\frac{2\int\left(\cosh\left(a+b\sqrt{c+dx}\right)(c+dx)^{5/2}-2c\cosh\left(a+b\sqrt{c+dx}\right)(c+dx)^{3/2}+c^2\cosh\left(a+b\sqrt{c+dx}\right)\sqrt{c+dx}\right)d\sqrt{}}{d^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{2\left(-\frac{120\cosh\left(a+b\sqrt{c+dx}\right)}{b^6}+\frac{120\sqrt{c+dx}\sinh\left(a+b\sqrt{c+dx}\right)}{b^5}-\frac{60(c+dx)\cosh\left(a+b\sqrt{c+dx}\right)}{b^4}+\frac{12c\cosh\left(a+b\sqrt{c+dx}\right)}{b^4}+\frac{20(c+dx)^{3/2}\sinh\left(a+\right.}{b^3}\right.}{}$$

input
```
Int[x^2*Cosh[a + b*Sqrt[c + d*x]],x]
```

output
```
(2*((-120*Cosh[a + b*Sqrt[c + d*x]])/b^6 + (12*c*Cosh[a + b*Sqrt[c + d*x]]
)/b^4 - (c^2*Cosh[a + b*Sqrt[c + d*x]])/b^2 - (60*(c + d*x)*Cosh[a + b*Sqr
t[c + d*x]])/b^4 + (6*c*(c + d*x)*Cosh[a + b*Sqrt[c + d*x]])/b^2 - (5*(c +
 d*x)^2*Cosh[a + b*Sqrt[c + d*x]])/b^2 + (120*Sqrt[c + d*x]*Sinh[a + b*Sqr
t[c + d*x]])/b^5 - (12*c*Sqrt[c + d*x]*Sinh[a + b*Sqrt[c + d*x]])/b^3 + (c
^2*Sqrt[c + d*x]*Sinh[a + b*Sqrt[c + d*x]])/b + (20*(c + d*x)^(3/2)*Sinh[a
 + b*Sqrt[c + d*x]])/b^3 - (2*c*(c + d*x)^(3/2)*Sinh[a + b*Sqrt[c + d*x]])
/b + ((c + d*x)^(5/2)*Sinh[a + b*Sqrt[c + d*x]])/b))/d^3
```

3.59.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 5888
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/Coefficient[u, x, 1]^(m + 1)   Subst[Int[(x - Coefficient[u, x
, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p
}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.59.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(310) = 620$.

Time = 0.18 (sec) , antiderivative size = 831, normalized size of antiderivative = 2.40

method	result
derivativedivides	$\frac{10a^4((a+b\sqrt{dx+c})\sinh(a+b\sqrt{dx+c})-\cosh(a+b\sqrt{dx+c}))}{b^4} - \frac{2a^5\sinh(a+b\sqrt{dx+c})}{b^4} - \frac{20a^3\left((a+b\sqrt{dx+c})^2\sinh(a+b\sqrt{dx+c})-2(a+b\sqrt{d}{b^4}$
default	$\frac{10a^4((a+b\sqrt{dx+c})\sinh(a+b\sqrt{dx+c})-\cosh(a+b\sqrt{dx+c}))}{b^4} - \frac{2a^5\sinh(a+b\sqrt{dx+c})}{b^4} - \frac{20a^3\left((a+b\sqrt{dx+c})^2\sinh(a+b\sqrt{dx+c})-2(a+b\sqrt{d}{b^4}$
parts	$\frac{2x^2\sinh(a+b\sqrt{dx+c})\sqrt{dx+c}}{db} - \frac{2x^2\cosh(a+b\sqrt{dx+c})}{d\,b^2} - 8\left(\frac{6a^2\left((a+b\sqrt{dx+c})^2\cosh(a+b\sqrt{dx+c})-2(a+b\sqrt{dx+c})\sinh(a+}{b^2}\right.$

input
```
int(x^2*cosh(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/d^3/b^2*(5/b^4*a^4*((a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-cosh(a+b
*(d*x+c)^(1/2)))-1/b^4*a^5*sinh(a+b*(d*x+c)^(1/2))-10/b^4*a^3*((a+b*(d*x+c
)^(1/2))^2*sinh(a+b*(d*x+c)^(1/2))-2*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^
(1/2))+2*sinh(a+b*(d*x+c)^(1/2)))+10/b^4*a^2*((a+b*(d*x+c)^(1/2))^3*sinh(a
+b*(d*x+c)^(1/2))-3*(a+b*(d*x+c)^(1/2))^2*cosh(a+b*(d*x+c)^(1/2))+6*(a+b*(
d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-6*cosh(a+b*(d*x+c)^(1/2)))-6/b^2*a^2
*c*((a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-cosh(a+b*(d*x+c)^(1/2)))+2
/b^2*a^3*c*sinh(a+b*(d*x+c)^(1/2))-5/b^4*a*((a+b*(d*x+c)^(1/2))^4*sinh(a+b
*(d*x+c)^(1/2))-4*(a+b*(d*x+c)^(1/2))^3*cosh(a+b*(d*x+c)^(1/2))+12*(a+b*(d
*x+c)^(1/2))^2*sinh(a+b*(d*x+c)^(1/2))-24*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*
x+c)^(1/2))+24*sinh(a+b*(d*x+c)^(1/2)))+6/b^2*a*c*((a+b*(d*x+c)^(1/2))^2*s
inh(a+b*(d*x+c)^(1/2))-2*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))+2*sin
h(a+b*(d*x+c)^(1/2)))+1/b^4*((a+b*(d*x+c)^(1/2))^5*sinh(a+b*(d*x+c)^(1/2))
-5*(a+b*(d*x+c)^(1/2))^4*cosh(a+b*(d*x+c)^(1/2))+20*(a+b*(d*x+c)^(1/2))^3*
sinh(a+b*(d*x+c)^(1/2))-60*(a+b*(d*x+c)^(1/2))^2*cosh(a+b*(d*x+c)^(1/2))+1
20*(a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-120*cosh(a+b*(d*x+c)^(1/2))
)-2/b^2*c*((a+b*(d*x+c)^(1/2))^3*sinh(a+b*(d*x+c)^(1/2))-3*(a+b*(d*x+c)^(1
/2))^2*cosh(a+b*(d*x+c)^(1/2))+6*(a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2
))-6*cosh(a+b*(d*x+c)^(1/2)))+c^2*((a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1
/2))-cosh(a+b*(d*x+c)^(1/2)))-c^2*a*sinh(a+b*(d*x+c)^(1/2)))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.30

$$\int x^2 \cosh \left(a + b\sqrt{c + dx}\right) \, dx$$
$$= \frac{2 \left((b^5 d^2 x^2 + 20 \, b^3 dx + 8 \, b^3 c + 120 \, b)\sqrt{dx + c} \sinh \left(\sqrt{dx + c} b + a\right) - (5 \, b^4 d^2 x^2 + 48 \, b^2 c + 4 \, (b^4 c + 15 \, b^2) dx\right.}{b^6 d^3}$$

input
```
integrate(x^2*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
2*((b^5*d^2*x^2 + 20*b^3*d*x + 8*b^3*c + 120*b)*sqrt(d*x + c)*sinh(sqrt(d*
x + c)*b + a) - (5*b^4*d^2*x^2 + 48*b^2*c + 4*(b^4*c + 15*b^2)*d*x + 120)*
cosh(sqrt(d*x + c)*b + a))/(b^6*d^3)
```

3.59.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.78

$$\int x^2 \cosh \left(a + b\sqrt{c + dx}\right) \, dx$$
$$= \begin{cases} \frac{x^3 \cosh(a)}{3} \\ \frac{x^3 \cosh(a+b\sqrt{c})}{3} \\ \frac{2x^2\sqrt{c+dx}\sinh(a+b\sqrt{c+dx})}{bd} - \frac{8cx\cosh(a+b\sqrt{c+dx})}{b^2d^2} - \frac{10x^2\cosh(a+b\sqrt{c+dx})}{b^2d} + \frac{16c\sqrt{c+dx}\sinh(a+b\sqrt{c+dx})}{b^3d^3} + \frac{40x\sqrt{c+dx}\sinh(a}{b^3d^2} \end{cases}$$

input
```
integrate(x**2*cosh(a+b*(d*x+c)**(1/2)),x)
```

output
```
Piecewise((x**3*cosh(a)/3, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x**3*cosh(a
 + b*sqrt(c))/3, Eq(d, 0)), (2*x**2*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x)
)/(b*d) - 8*c*x*cosh(a + b*sqrt(c + d*x))/(b**2*d**2) - 10*x**2*cosh(a + b
*sqrt(c + d*x))/(b**2*d) + 16*c*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b
**3*d**3) + 40*x*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b**3*d**2) - 96*
c*cosh(a + b*sqrt(c + d*x))/(b**4*d**3) - 120*x*cosh(a + b*sqrt(c + d*x))/
(b**4*d**2) + 240*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b**5*d**3) - 24
0*cosh(a + b*sqrt(c + d*x))/(b**6*d**3), True))
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.40

$$\int x^2 \cosh \left(a + b\sqrt{c + dx}\right) \, dx$$
$$= \frac{2\,d^3x^3\cosh\left(\sqrt{dx+c}b+a\right) + \left(\frac{c^3e^{\left(\sqrt{dx+c}b+a\right)}}{b} + \frac{c^3e^{\left(-\sqrt{dx+c}b-a\right)}}{b} - \frac{3\left((dx+c)b^2e^a - 2\sqrt{dx+c}be^a + 2e^a\right)c^2e^{\left(\sqrt{dx+c}b\right)}}{b^3} - \frac{3\left((dx\right.}{}}{}$$

input
```
integrate(x^2*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
1/6*(2*d^3*x^3*cosh(sqrt(d*x + c)*b + a) + (c^3*e^(sqrt(d*x + c)*b + a)/b
+ c^3*e^(-sqrt(d*x + c)*b - a)/b - 3*((d*x + c)*b^2*e^a - 2*sqrt(d*x + c)*
b*e^a + 2*e^a)*c^2*e^(sqrt(d*x + c)*b)/b^3 - 3*((d*x + c)*b^2 + 2*sqrt(d*x
 + c)*b + 2)*c^2*e^(-sqrt(d*x + c)*b - a)/b^3 + 3*((d*x + c)^2*b^4*e^a - 4
*(d*x + c)^(3/2)*b^3*e^a + 12*(d*x + c)*b^2*e^a - 24*sqrt(d*x + c)*b*e^a +
 24*e^a)*c*e^(sqrt(d*x + c)*b)/b^5 + 3*((d*x + c)^2*b^4 + 4*(d*x + c)^(3/2
)*b^3 + 12*(d*x + c)*b^2 + 24*sqrt(d*x + c)*b + 24)*c*e^(-sqrt(d*x + c)*b
- a)/b^5 - ((d*x + c)^3*b^6*e^a - 6*(d*x + c)^(5/2)*b^5*e^a + 30*(d*x + c)
^2*b^4*e^a - 120*(d*x + c)^(3/2)*b^3*e^a + 360*(d*x + c)*b^2*e^a - 720*sqr
t(d*x + c)*b*e^a + 720*e^a)*e^(sqrt(d*x + c)*b)/b^7 - ((d*x + c)^3*b^6 + 6
*(d*x + c)^(5/2)*b^5 + 30*(d*x + c)^2*b^4 + 120*(d*x + c)^(3/2)*b^3 + 360*
(d*x + c)*b^2 + 720*sqrt(d*x + c)*b + 720)*e^(-sqrt(d*x + c)*b - a)/b^7)*b
)/d^3
```

3.59.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(310) = 620$.

Time = 0.29 (sec) , antiderivative size = 915, normalized size of antiderivative = 2.64

$$\int x^2 \cosh \left(a+b\sqrt{c+dx}\right) \, dx$$
$$= \frac{\frac{\left(\left(\sqrt{dx+c}b+a\right)b^4c^2-ab^4c^2-2\left(\sqrt{dx+c}b+a\right)^3b^2c+6\left(\sqrt{dx+c}b+a\right)^2ab^2c-6\left(\sqrt{dx+c}b+a\right)a^2b^2c+2a^3b^2c-b^4c^2+\left(\sqrt{dx+c}b+a\right)^5-5\left(\sqrt{dx+c}b+a\right)^4a}{}}{}$$

input
```
integrate(x^2*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
(((sqrt(d*x + c)*b + a)*b^4*c^2 - a*b^4*c^2 - 2*(sqrt(d*x + c)*b + a)^3*b^
2*c + 6*(sqrt(d*x + c)*b + a)^2*a*b^2*c - 6*(sqrt(d*x + c)*b + a)*a^2*b^2*
c + 2*a^3*b^2*c - b^4*c^2 + (sqrt(d*x + c)*b + a)^5 - 5*(sqrt(d*x + c)*b +
 a)^4*a + 10*(sqrt(d*x + c)*b + a)^3*a^2 - 10*(sqrt(d*x + c)*b + a)^2*a^3
+ 5*(sqrt(d*x + c)*b + a)*a^4 - a^5 + 6*(sqrt(d*x + c)*b + a)^2*b^2*c - 12
*(sqrt(d*x + c)*b + a)*a*b^2*c + 6*a^2*b^2*c - 5*(sqrt(d*x + c)*b + a)^4 +
 20*(sqrt(d*x + c)*b + a)^3*a - 30*(sqrt(d*x + c)*b + a)^2*a^2 + 20*(sqrt(
d*x + c)*b + a)*a^3 - 5*a^4 - 12*(sqrt(d*x + c)*b + a)*b^2*c + 12*a*b^2*c
+ 20*(sqrt(d*x + c)*b + a)^3 - 60*(sqrt(d*x + c)*b + a)^2*a + 60*(sqrt(d*x
 + c)*b + a)*a^2 - 20*a^3 + 12*b^2*c - 60*(sqrt(d*x + c)*b + a)^2 + 120*(s
qrt(d*x + c)*b + a)*a - 60*a^2 + 120*sqrt(d*x + c)*b - 120)*e^(sqrt(d*x +
c)*b + a)/(b^5*d^2) - ((sqrt(d*x + c)*b + a)*b^4*c^2 - a*b^4*c^2 - 2*(sqrt
(d*x + c)*b + a)^3*b^2*c + 6*(sqrt(d*x + c)*b + a)^2*a*b^2*c - 6*(sqrt(d*x
 + c)*b + a)*a^2*b^2*c + 2*a^3*b^2*c + b^4*c^2 + (sqrt(d*x + c)*b + a)^5 -
 5*(sqrt(d*x + c)*b + a)^4*a + 10*(sqrt(d*x + c)*b + a)^3*a^2 - 10*(sqrt(d
*x + c)*b + a)^2*a^3 + 5*(sqrt(d*x + c)*b + a)*a^4 - a^5 - 6*(sqrt(d*x + c
)*b + a)^2*b^2*c + 12*(sqrt(d*x + c)*b + a)*a*b^2*c - 6*a^2*b^2*c + 5*(sqr
t(d*x + c)*b + a)^4 - 20*(sqrt(d*x + c)*b + a)^3*a + 30*(sqrt(d*x + c)*b +
 a)^2*a^2 - 20*(sqrt(d*x + c)*b + a)*a^3 + 5*a^4 - 12*(sqrt(d*x + c)*b + a
)*b^2*c + 12*a*b^2*c + 20*(sqrt(d*x + c)*b + a)^3 - 60*(sqrt(d*x + c)*b...
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh \left(a+b\sqrt{c+dx}\right) \, dx=\int x^2\cosh\left(a+b\,\sqrt{c+d\,x}\right) \,dx$$

input
```
int(x^2*cosh(a + b*(c + d*x)^(1/2)),x)
```

output
```
int(x^2*cosh(a + b*(c + d*x)^(1/2)), x)
```

3.60 $\int x \cosh\left(a+b\sqrt{c+dx}\right)\,dx$

3.60.1 Optimal result

Integrand size = 16, antiderivative size = 167

$$\int x\cosh\left(a+b\sqrt{c+dx}\right)\,dx=-\frac{12\cosh\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{2c\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{6(c+dx)\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{12\sqrt{c+dx}\sinh\left(a+b\sqrt{c+dx}\right)}{b^3d^2}-\frac{2c\sqrt{c+dx}\sinh\left(a+b\sqrt{c+dx}\right)}{bd^2}+\frac{2(c+dx)^{3/2}\sinh\left(a+b\sqrt{c+dx}\right)}{bd^2}$$

output
```
-12*cosh(a+b*(d*x+c)^(1/2))/b^4/d^2+2*c*cosh(a+b*(d*x+c)^(1/2))/b^2/d^2-6*
(d*x+c)*cosh(a+b*(d*x+c)^(1/2))/b^2/d^2+2*(d*x+c)^(3/2)*sinh(a+b*(d*x+c)^(
1/2))/b/d^2+12*sinh(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^3/d^2-2*c*sinh(a+b*
(d*x+c)^(1/2))*(d*x+c)^(1/2)/b/d^2
```

3.60.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.43

$$\int x\cosh\left(a+b\sqrt{c+dx}\right)\,dx = \frac{-2(6+b^2(2c+3dx))\cosh\left(a+b\sqrt{c+dx}\right)+2b\sqrt{c+dx}(6+b^2dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

input
```
Integrate[x*Cosh[a + b*Sqrt[c + d*x]],x]
```

output
```
(-2*(6 + b^2*(2*c + 3*d*x))*Cosh[a + b*Sqrt[c + d*x]] + 2*b*Sqrt[c + d*x]*
(6 + b^2*d*x)*Sinh[a + b*Sqrt[c + d*x]])/(b^4*d^2)
```

3.60.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5888, 25, 7267, 5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh\left(a+b\sqrt{c+dx}\right)\,dx$$

$$\Big\downarrow \text{5888}$$

$$\frac{\int dx\cosh\left(a+b\sqrt{c+dx}\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -dx\cosh\left(a+b\sqrt{c+dx}\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \text{7267}$$

$$-\frac{2\int -dx\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{5810}$$

$$-\frac{2\int \left(c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)-(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)\right)d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(\frac{6\cosh\left(a+b\sqrt{c+dx}\right)}{b^4}-\frac{6\sqrt{c+dx}\sinh\left(a+b\sqrt{c+dx}\right)}{b^3}+\frac{3(c+dx)\cosh\left(a+b\sqrt{c+dx}\right)}{b^2}-\frac{c\cosh\left(a+b\sqrt{c+dx}\right)}{b^2}-\frac{(c+dx)^{3/2}\sinh\left(a+b\sqrt{c+dx}\right)}{b}\right.}{d^2}$$

input
```
Int[x*Cosh[a + b*Sqrt[c + d*x]],x]
```

output
```
(-2*((6*Cosh[a + b*Sqrt[c + d*x]])/b^4 - (c*Cosh[a + b*Sqrt[c + d*x]])/b^2
 + (3*(c + d*x)*Cosh[a + b*Sqrt[c + d*x]])/b^2 - (6*Sqrt[c + d*x]*Sinh[a +
 b*Sqrt[c + d*x]])/b^3 + (c*Sqrt[c + d*x]*Sinh[a + b*Sqrt[c + d*x]])/b - (
(c + d*x)^(3/2)*Sinh[a + b*Sqrt[c + d*x]])/b))/d^2
```

3.60.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 5888
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/Coefficient[u, x, 1]^(m + 1)   Subst[Int[(x - Coefficient[u, x
, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p
}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.60.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(149) = 298$.

Time = 0.12 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.80

method	result
parts	$\frac{2x\sqrt{dx+c}\,\sinh\left(a+b\sqrt{dx+c}\right)}{db}-\frac{2x\cosh\left(a+b\sqrt{dx+c}\right)}{d\,b^{2}}-\frac{2\left(\frac{2\left(a+b\sqrt{dx+c}\right)^{2}\cosh\left(a+b\sqrt{dx+c}\right)-4\left(a+b\sqrt{dx+c}\right)\sinh\left(a+b\sqrt{dx\cdots\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{6a^{2}\left(\left(a+b\sqrt{dx+c}\right)\sinh\left(a+b\sqrt{dx+c}\right)-\cosh\left(a+b\sqrt{dx+c}\right)\right)}{b^{2}}-\frac{2a^{3}\sinh\left(a+b\sqrt{dx+c}\right)}{b^{2}}-\frac{6a\left(\left(a+b\sqrt{dx+c}\right)^{2}\sinh\left(a+b\sqrt{dx+c}\right)-2\left(a+b\sqrt{dx+\cdots\right.\right.}{b^{2}}}{}$
default	$\frac{\frac{6a^{2}\left(\left(a+b\sqrt{dx+c}\right)\sinh\left(a+b\sqrt{dx+c}\right)-\cosh\left(a+b\sqrt{dx+c}\right)\right)}{b^{2}}-\frac{2a^{3}\sinh\left(a+b\sqrt{dx+c}\right)}{b^{2}}-\frac{6a\left(\left(a+b\sqrt{dx+c}\right)^{2}\sinh\left(a+b\sqrt{dx+c}\right)-2\left(a+b\sqrt{dx+\cdots\right.\right.}{b^{2}}}{}$

input
```
int(x*cosh(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/d/b*x*(d*x+c)^(1/2)*sinh(a+b*(d*x+c)^(1/2))-2/d/b^2*x*cosh(a+b*(d*x+c)^(
1/2))-2/d/b^2*(2/d/b^2*((a+b*(d*x+c)^(1/2))^2*cosh(a+b*(d*x+c)^(1/2))-2*(a
+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))+2*cosh(a+b*(d*x+c)^(1/2))-a*((a+
b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+b*(d*x+c)^(1/2))))-2*a/d/b
^2*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+b*(d*x+c)^(1/2))-a*
cosh(a+b*(d*x+c)^(1/2)))-2/d/b^2*((a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/
2))-cosh(a+b*(d*x+c)^(1/2))-a*sinh(a+b*(d*x+c)^(1/2))))
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.41

$$\int x\cosh\left(a+b\sqrt{c+dx}\right)\,dx = \frac{2\left(\left(b^{3}dx+6\,b\right)\sqrt{dx+c}\sinh\left(\sqrt{dx+c}b+a\right)-\left(3\,b^{2}dx+2\,b^{2}c+6\right)\cosh\left(\sqrt{dx+c}b+a\right)\right)}{b^{4}d^{2}}$$

input
```
integrate(x*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
2*((b^3*d*x + 6*b)*sqrt(d*x + c)*sinh(sqrt(d*x + c)*b + a) - (3*b^2*d*x +
2*b^2*c + 6)*cosh(sqrt(d*x + c)*b + a))/(b^4*d^2)
```

3.60.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int x\cosh\left(a+b\sqrt{c+dx}\right)\,dx$$
$$= \begin{cases} \frac{x^2\cosh(a)}{2} \\ \frac{x^2\cosh(a+b\sqrt{c})}{2} \\ \frac{2x\sqrt{c+dx}\sinh(a+b\sqrt{c+dx})}{bd} - \frac{4c\cosh(a+b\sqrt{c+dx})}{b^2d^2} - \frac{6x\cosh(a+b\sqrt{c+dx})}{b^2d} + \frac{12\sqrt{c+dx}\sinh(a+b\sqrt{c+dx})}{b^3d^2} - \frac{12\cosh(a+b\sqrt{c+dx})}{b^4d^2} \end{cases}$$

input
```
integrate(x*cosh(a+b*(d*x+c)**(1/2)),x)
```

output
```
Piecewise((x**2*cosh(a)/2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x**2*cosh(a
 + b*sqrt(c))/2, Eq(d, 0)), (2*x*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(
b*d) - 4*c*cosh(a + b*sqrt(c + d*x))/(b**2*d**2) - 6*x*cosh(a + b*sqrt(c +
 d*x))/(b**2*d) + 12*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b**3*d**2) -
 12*cosh(a + b*sqrt(c + d*x))/(b**4*d**2), True))
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.74

$$\int x\cosh\left(a+b\sqrt{c+dx}\right)\,dx$$
$$= \frac{2\,d^2x^2\cosh\left(\sqrt{dx+c}b+a\right) - \left(\frac{c^2e^{(\sqrt{dx+c}b+a)}}{b} + \frac{c^2e^{(-\sqrt{dx+c}b-a)}}{b} - \frac{2\left((dx+c)b^2e^a-2\sqrt{dx+c}be^a+2\,e^a\right)ce^{(\sqrt{dx+c}b)}}{b^3} - \frac{2\,((dx+\cdots}{}\right.}{}$$

input
```
integrate(x*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
1/4*(2*d^2*x^2*cosh(sqrt(d*x + c)*b + a) - (c^2*e^(sqrt(d*x + c)*b + a)/b
+ c^2*e^(-sqrt(d*x + c)*b - a)/b - 2*((d*x + c)*b^2*e^a - 2*sqrt(d*x + c)*
b*e^a + 2*e^a)*c*e^(sqrt(d*x + c)*b)/b^3 - 2*((d*x + c)*b^2 + 2*sqrt(d*x +
 c)*b + 2)*c*e^(-sqrt(d*x + c)*b - a)/b^3 + ((d*x + c)^2*b^4*e^a - 4*(d*x
+ c)^(3/2)*b^3*e^a + 12*(d*x + c)*b^2*e^a - 24*sqrt(d*x + c)*b*e^a + 24*e^
a)*e^(sqrt(d*x + c)*b)/b^5 + ((d*x + c)^2*b^4 + 4*(d*x + c)^(3/2)*b^3 + 12
*(d*x + c)*b^2 + 24*sqrt(d*x + c)*b + 24)*e^(-sqrt(d*x + c)*b - a)/b^5)*b)
/d^2
```

3.60.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(149) = 298$.

Time = 0.26 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.80

$$\int x \cosh \left(a+b\sqrt{c+dx}\right)\, dx =$$

$$-\frac{\frac{\left(\left(\sqrt{dx+c}b+a\right)b^2c-ab^2c-\left(\sqrt{dx+c}b+a\right)^3+3\left(\sqrt{dx+c}b+a\right)^2a-3\left(\sqrt{dx+c}b+a\right)a^2+a^3-b^2c+3\left(\sqrt{dx+c}b+a\right)^2-6\left(\sqrt{dx+c}b+a\right)a+3a^2-6\sqrt{dx+}\right.}{b^3d}}{}$$

input
```
integrate(x*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
-(((sqrt(d*x + c)*b + a)*b^2*c - a*b^2*c - (sqrt(d*x + c)*b + a)^3 + 3*(sq
rt(d*x + c)*b + a)^2*a - 3*(sqrt(d*x + c)*b + a)*a^2 + a^3 - b^2*c + 3*(sq
rt(d*x + c)*b + a)^2 - 6*(sqrt(d*x + c)*b + a)*a + 3*a^2 - 6*sqrt(d*x + c)
*b + 6)*e^(sqrt(d*x + c)*b + a)/(b^3*d) - ((sqrt(d*x + c)*b + a)*b^2*c - a
*b^2*c - (sqrt(d*x + c)*b + a)^3 + 3*(sqrt(d*x + c)*b + a)^2*a - 3*(sqrt(d
*x + c)*b + a)*a^2 + a^3 + b^2*c - 3*(sqrt(d*x + c)*b + a)^2 + 6*(sqrt(d*x
 + c)*b + a)*a - 3*a^2 - 6*sqrt(d*x + c)*b - 6)*e^(-sqrt(d*x + c)*b - a)/(
b^3*d))/(b*d)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int x \cosh \left(a+b\sqrt{c+dx}\right)\, dx = \int x \cosh\left(a+b\sqrt{c+d\,x}\right) \,dx$$

input
```
int(x*cosh(a + b*(c + d*x)^(1/2)),x)
```

output
```
int(x*cosh(a + b*(c + d*x)^(1/2)), x)
```

3.61 $\int \cosh \left(a+b\sqrt{c+dx}\right) \, dx$

3.61.1 Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \cosh \left(a+b\sqrt{c+dx}\right) \, dx = -\frac{2\cosh \left(a+b\sqrt{c+dx}\right)}{b^2 d}+\frac{2\sqrt{c+dx}\sinh \left(a+b\sqrt{c+dx}\right)}{bd}$$

output
```
-2*cosh(a+b*(d*x+c)^(1/2))/b^2/d+2*sinh(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b
/d
```

3.61.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \cosh \left(a+b\sqrt{c+dx}\right) \, dx = \frac{2\left(-\cosh \left(a+b\sqrt{c+dx}\right)+b\sqrt{c+dx}\sinh \left(a+b\sqrt{c+dx}\right)\right)}{b^2 d}$$

input
```
Integrate[Cosh[a + b*Sqrt[c + d*x]],x]
```

output
```
(2*(-Cosh[a + b*Sqrt[c + d*x]] + b*Sqrt[c + d*x]*Sinh[a + b*Sqrt[c + d*x]]
))/(b^2*d)
```

3.61.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5834, 5828, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh \left(a+b\sqrt{c+dx}\right) dx$$

$$\Big\downarrow \; 5834$$

$$\frac{\int \cosh \left(a+b\sqrt{c+dx}\right) d(c+dx)}{d}$$

$$\Big\downarrow \; 5828$$

$$\frac{2\int \sqrt{c+dx}\cosh \left(a+b\sqrt{c+dx}\right) d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\int \sqrt{c+dx}\sin \left(ia+ib\sqrt{c+dx}+\frac{\pi}{2}\right) d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \; 3777$$

$$\frac{2\left(\frac{\sqrt{c+dx}\sinh\left(a+b\sqrt{c+dx}\right)}{b}-\frac{i\int -i\sinh\left(a+b\sqrt{c+dx}\right)d\sqrt{c+dx}}{b}\right)}{d}$$

$$\Big\downarrow \; 26$$

$$\frac{2\left(\frac{\sqrt{c+dx}\sinh\left(a+b\sqrt{c+dx}\right)}{b}-\frac{\int \sinh\left(a+b\sqrt{c+dx}\right)d\sqrt{c+dx}}{b}\right)}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\left(\frac{\sqrt{c+dx}\sinh\left(a+b\sqrt{c+dx}\right)}{b}-\frac{\int -i\sin\left(ia+ib\sqrt{c+dx}\right)d\sqrt{c+dx}}{b}\right)}{d}$$

$$\Big\downarrow \; 26$$

$$\frac{2\left(\frac{\sqrt{c+dx}\sinh\left(a+b\sqrt{c+dx}\right)}{b}+\frac{i\int \sin\left(ia+ib\sqrt{c+dx}\right)d\sqrt{c+dx}}{b}\right)}{d}$$

$$\Big\downarrow \; 3118$$

$$\frac{2\left(\frac{\sqrt{c+dx}\sinh\left(a+b\sqrt{c+dx}\right)}{b}-\frac{\cosh\left(a+b\sqrt{c+dx}\right)}{b^2}\right)}{d}$$

input
```
Int[Cosh[a + b*Sqrt[c + d*x]],x]
```

output
```
(2*(-(Cosh[a + b*Sqrt[c + d*x]]/b^2) + (Sqrt[c + d*x]*Sinh[a + b*Sqrt[c +
d*x]])/b))/d
```

3.61.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5828
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.), x_Symbol] :> Modul
e[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Cosh[c + d*x^(k
*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d}, x] && FractionQ[n] &&
IntegerQ[p]
```

rule 5834
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.), x_Symbol] :> Simp[
1/Coefficient[u, x, 1]   Subst[Int[(a + b*Cosh[c + d*x^n])^p, x], x, u], x]
 /; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

3.61.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{2(a+b\sqrt{dx+c})\sinh(a+b\sqrt{dx+c})-2\cosh(a+b\sqrt{dx+c})-2a\sinh(a+b\sqrt{dx+c})}{b^2d}$	63
default	$\frac{2(a+b\sqrt{dx+c})\sinh(a+b\sqrt{dx+c})-2\cosh(a+b\sqrt{dx+c})-2a\sinh(a+b\sqrt{dx+c})}{b^2d}$	63

input
```
int(cosh(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/d/b^2*((a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-cosh(a+b*(d*x+c)^(1/2
))-a*sinh(a+b*(d*x+c)^(1/2)))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \cosh\left(a+b\sqrt{c+dx}\right)\,dx=\frac{2\left(\sqrt{dx+c}b\sinh\left(\sqrt{dx+c}b+a\right)-\cosh\left(\sqrt{dx+c}b+a\right)\right)}{b^2d}$$

input
```
integrate(cosh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
2*(sqrt(d*x + c)*b*sinh(sqrt(d*x + c)*b + a) - cosh(sqrt(d*x + c)*b + a))/
(b^2*d)
```

3.61.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int \cosh\left(a+b\sqrt{c+dx}\right)\,dx = \begin{cases} x\cosh(a) & \text{for } b=0\wedge(b=0\vee d=0) \\ x\cosh\left(a+b\sqrt{c}\right) & \text{for } d=0 \\ \frac{2\sqrt{c+dx}\sinh\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(a+b*(d*x+c)**(1/2)),x)
```

output
```
Piecewise((x*cosh(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*cosh(a + b*sqr
t(c)), Eq(d, 0)), (2*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b*d) - 2*cos
h(a + b*sqrt(c + d*x))/(b**2*d), True))
```

3.61.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(48) = 96$.

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.04

$$\int \cosh\left(a+b\sqrt{c+dx}\right)\,dx = -\frac{b\left(\frac{\left((dx+c)b^2e^a-2\sqrt{dx+c}be^a+2e^a\right)e^{\left(\sqrt{dx+c}b\right)}}{b^3}+\frac{\left((dx+c)b^2+2\sqrt{dx+c}b+2\right)e^{\left(-\sqrt{dx+c}b-a\right)}}{b^3}\right)-2\,(dx+c)\cosh\left(\sqrt{dx+c}b+a\right)}{2\,d}$$

input
```
integrate(cosh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
-1/2*(b*(((d*x + c)*b^2*e^a - 2*sqrt(d*x + c)*b*e^a + 2*e^a)*e^(sqrt(d*x +
 c)*b)/b^3 + ((d*x + c)*b^2 + 2*sqrt(d*x + c)*b + 2)*e^(-sqrt(d*x + c)*b -
 a)/b^3) - 2*(d*x + c)*cosh(sqrt(d*x + c)*b + a))/d
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int \cosh\left(a+b\sqrt{c+dx}\right)\,dx = \frac{\left(\sqrt{dx+c}b-1\right)e^{\left(\sqrt{dx+c}b+a\right)}}{b^2d}-\frac{\left(\sqrt{dx+c}b+1\right)e^{\left(-\sqrt{dx+c}b-a\right)}}{b^2d}$$

input
```
integrate(cosh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
(sqrt(d*x + c)*b - 1)*e^(sqrt(d*x + c)*b + a)/(b^2*d) - (sqrt(d*x + c)*b +
 1)*e^(-sqrt(d*x + c)*b - a)/(b^2*d)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \cosh \left(a+b\sqrt{c+dx}\right) \, dx = -\frac{2 \left(\cosh\left(a+b\sqrt{c+d\,x}\right)-b\sinh\left(a+b\sqrt{c+d\,x}\right)\,\sqrt{c+d\,x}\right)}{b^2\,d}$$

input
```
int(cosh(a + b*(c + d*x)^(1/2)),x)
```

output
```
-(2*(cosh(a + b*(c + d*x)^(1/2)) - b*sinh(a + b*(c + d*x)^(1/2))*(c + d*x)
^(1/2)))/(b^2*d)
```

3.62 $\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x}\,dx$

3.62.1 Optimal result

Integrand size = 18, antiderivative size = 124

$$\begin{aligned}\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x}\,dx &= \cosh\left(a+b\sqrt{c}\right)\operatorname{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)\\ &\quad+\cosh\left(a-b\sqrt{c}\right)\operatorname{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\\ &\quad-\sinh\left(a+b\sqrt{c}\right)\operatorname{Shi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)\\ &\quad+\sinh\left(a-b\sqrt{c}\right)\operatorname{Shi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\end{aligned}$$

output
```
Chi(b*(c^(1/2)+(d*x+c)^(1/2)))*cosh(a-b*c^(1/2))+Chi(b*(c^(1/2)-(d*x+c)^(1
/2)))*cosh(a+b*c^(1/2))+Shi(b*(c^(1/2)+(d*x+c)^(1/2)))*sinh(a-b*c^(1/2))-S
hi(b*(c^(1/2)-(d*x+c)^(1/2)))*sinh(a+b*c^(1/2))
```

3.62.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x}\,dx &= \frac{1}{2}e^{-a-b\sqrt{c}}\Big(\text{ExpIntegralEi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)\\ &\quad+e^{2\left(a+b\sqrt{c}\right)}\,\text{ExpIntegralEi}\left(b\left(-\sqrt{c}+\sqrt{c+dx}\right)\right)\\ &\quad+e^{2b\sqrt{c}}\,\text{ExpIntegralEi}\left(-b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\\ &\quad+e^{2a}\,\text{ExpIntegralEi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\Big)\end{aligned}$$

input
```
Integrate[Cosh[a + b*Sqrt[c + d*x]]/x,x]
```

output
```
(E^(-a - b*Sqrt[c])*(ExpIntegralEi[b*(Sqrt[c] - Sqrt[c + d*x])] + E^(2*(a
+ b*Sqrt[c]))*ExpIntegralEi[b*(-Sqrt[c] + Sqrt[c + d*x])] + E^(2*b*Sqrt[c]
)*ExpIntegralEi[-(b*(Sqrt[c] + Sqrt[c + d*x]))] + E^(2*a)*ExpIntegralEi[b*
(Sqrt[c] + Sqrt[c + d*x])]))/2
```

3.62.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5888, 25, 7267, 5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x}\,dx$$

$$\Big\downarrow \text{5888}$$

$$\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{dx}d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{\cosh\left(a+b\sqrt{c+dx}\right)}{dx}d(c+dx)$$

$$\Big\downarrow \text{7267}$$

$$-2\int -\frac{\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{dx}d\sqrt{c+dx}$$

$$\Big\downarrow \text{5816}$$

$$-2\int \left(\frac{\cosh\left(a+b\sqrt{c+dx}\right)}{2\left(-c+\sqrt{c}-dx\right)}-\frac{\cosh\left(a+b\sqrt{c+dx}\right)}{2\left(\sqrt{c}+\sqrt{c+dx}\right)}\right)d\sqrt{c+dx}$$

$$\Big\downarrow \text{2009}$$

$$-2\left(-\frac{1}{2}\cosh\left(a+b\sqrt{c}\right)\text{Chi}\left(b\sqrt{c}-b\sqrt{c+dx}\right)-\frac{1}{2}\cosh\left(a-b\sqrt{c}\right)\text{Chi}\left(\sqrt{c}b+\sqrt{c+dx}b\right)+\frac{1}{2}\sinh\left(a+b\sqrt{c}\right)\text{Shi}\right.$$

input
```
Int[Cosh[a + b*Sqrt[c + d*x]]/x,x]
```

output
```
-2*(-1/2*(Cosh[a + b*Sqrt[c]]*CoshIntegral[b*Sqrt[c] - b*Sqrt[c + d*x]]) -
 (Cosh[a - b*Sqrt[c]]*CoshIntegral[b*Sqrt[c] + b*Sqrt[c + d*x]])/2 + (Sinh
[a + b*Sqrt[c]]*SinhIntegral[b*Sqrt[c] - b*Sqrt[c + d*x]])/2 - (Sinh[a - b
*Sqrt[c]]*SinhIntegral[b*Sqrt[c] + b*Sqrt[c + d*x]])/2)
```

3.62.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

rule 5888
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/Coefficient[u, x, 1]^(m + 1)   Subst[Int[(x - Coefficient[u, x
, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p
}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.62.4 Maple [F]

$$\int \frac{\cosh\left(a+b\sqrt{dx+c}\right)}{x}dx$$

input
```
int(cosh(a+b*(d*x+c)^(1/2))/x,x)
```

output
```
int(cosh(a+b*(d*x+c)^(1/2))/x,x)
```

3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(102) = 204$.

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.75

$$\begin{aligned}
\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x}\,dx
&= \frac{1}{2}\left(\mathrm{Ei}\left(\sqrt{dx+c}b-\sqrt{b^2c}\right)+\mathrm{Ei}\left(-\sqrt{dx+c}b+\sqrt{b^2c}\right)\right)\cosh\left(a+\sqrt{b^2c}\right)\\
&\quad+\frac{1}{2}\left(\mathrm{Ei}\left(\sqrt{dx+c}b+\sqrt{b^2c}\right)+\mathrm{Ei}\left(-\sqrt{dx+c}b-\sqrt{b^2c}\right)\right)\cosh\left(-a+\sqrt{b^2c}\right)\\
&\quad+\frac{1}{2}\left(\mathrm{Ei}\left(\sqrt{dx+c}b-\sqrt{b^2c}\right)-\mathrm{Ei}\left(-\sqrt{dx+c}b+\sqrt{b^2c}\right)\right)\sinh\left(a+\sqrt{b^2c}\right)\\
&\quad-\frac{1}{2}\left(\mathrm{Ei}\left(\sqrt{dx+c}b+\sqrt{b^2c}\right)-\mathrm{Ei}\left(-\sqrt{dx+c}b-\sqrt{b^2c}\right)\right)\sinh\left(-a+\sqrt{b^2c}\right)
\end{aligned}$$

input
```
integrate(cosh(a+b*(d*x+c)^(1/2))/x,x, algorithm="fricas")
```

output
```
1/2*(Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) + Ei(-sqrt(d*x + c)*b + sqrt(b^2*c)
))*cosh(a + sqrt(b^2*c)) + 1/2*(Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) + Ei(-sq
rt(d*x + c)*b - sqrt(b^2*c)))*cosh(-a + sqrt(b^2*c)) + 1/2*(Ei(sqrt(d*x +
c)*b - sqrt(b^2*c)) - Ei(-sqrt(d*x + c)*b + sqrt(b^2*c)))*sinh(a + sqrt(b^
2*c)) - 1/2*(Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) - Ei(-sqrt(d*x + c)*b - sqr
t(b^2*c)))*sinh(-a + sqrt(b^2*c))
```

3.62.6 Sympy [F]

$$\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x}\,dx = \int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x}\,dx$$

input
```
integrate(cosh(a+b*(d*x+c)**(1/2))/x,x)
```

output
```
Integral(cosh(a + b*sqrt(c + d*x))/x, x)
```

3.62.7 Maxima [F]

$$\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x}\,dx = \int \frac{\cosh\left(\sqrt{dx+c}b+a\right)}{x}\,dx$$

input
```
integrate(cosh(a+b*(d*x+c)^(1/2))/x,x, algorithm="maxima")
```

output
```
integrate(cosh(sqrt(d*x + c)*b + a)/x, x)
```

3.62.8 Giac [F]

$$\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x}\,dx = \int \frac{\cosh\left(\sqrt{dx+c}b+a\right)}{x}\,dx$$

input
```
integrate(cosh(a+b*(d*x+c)^(1/2))/x,x, algorithm="giac")
```

output
```
integrate(cosh(sqrt(d*x + c)*b + a)/x, x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x}\,dx=\int \frac{\cosh\left(a+b\,\sqrt{c+d\,x}\right)}{x}\,dx$$

input
```
int(cosh(a + b*(c + d*x)^(1/2))/x,x)
```

output
```
int(cosh(a + b*(c + d*x)^(1/2))/x, x)
```

3.63 $\int \frac{\cosh(a+b\sqrt{c+dx})}{x^2} dx$

3.63.1 Optimal result

Integrand size = 18, antiderivative size = 182

$$\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x^2} dx = -\frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x} - \frac{bd\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\sinh\left(a-b\sqrt{c}\right)}{2\sqrt{c}} + \frac{bd\text{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)\sinh\left(a+b\sqrt{c}\right)}{2\sqrt{c}} - \frac{bd\cosh\left(a+b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)}{2\sqrt{c}} - \frac{bd\cosh\left(a-b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}}$$

output
```
-cosh(a+b*(d*x+c)^(1/2))/x-1/2*b*d*cosh(a+b*c^(1/2))*Shi(b*(c^(1/2)-(d*x+c
)^(1/2)))/c^(1/2)-1/2*b*d*cosh(a-b*c^(1/2))*Shi(b*(c^(1/2)+(d*x+c)^(1/2)))
/c^(1/2)-1/2*b*d*Chi(b*(c^(1/2)+(d*x+c)^(1/2)))*sinh(a-b*c^(1/2))/c^(1/2)+
1/2*b*d*Chi(b*(c^(1/2)-(d*x+c)^(1/2)))*sinh(a+b*c^(1/2))/c^(1/2)
```

3.63.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.12

$$\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx =$$
$$-\frac{e^{-a}\left(2\sqrt{c}e^{-b\sqrt{c+dx}}+2\sqrt{c}e^{2a+b\sqrt{c+dx}}+bde^{-b\sqrt{c}}x\,\text{ExpIntegralEi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)-bde^{2a+b\sqrt{c}}x\,\text{ExpIn}\right.}{}$$

input
```
Integrate[Cosh[a + b*Sqrt[c + d*x]]/x^2,x]
```

output
```
-1/4*((2*Sqrt[c])/E^(b*Sqrt[c + d*x]) + 2*Sqrt[c]*E^(2*a + b*Sqrt[c + d*x]
) + (b*d*x*ExpIntegralEi[b*(Sqrt[c] - Sqrt[c + d*x])])/E^(b*Sqrt[c]) - b*d
*E^(2*a + b*Sqrt[c])*x*ExpIntegralEi[b*(-Sqrt[c] + Sqrt[c + d*x])] - b*d*E
^(b*Sqrt[c])*x*ExpIntegralEi[-(b*(Sqrt[c] + Sqrt[c + d*x]))] + b*d*E^(2*a
- b*Sqrt[c])*x*ExpIntegralEi[b*(Sqrt[c] + Sqrt[c + d*x])])/(Sqrt[c]*E^a*x)
```

3.63.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5888, 7267, 5812, 5803, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x^2}dx$$
$$\Big\downarrow \text{5888}$$
$$d\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{d^2x^2}d(c+dx)$$
$$\Big\downarrow \text{7267}$$
$$2d\int \frac{\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{d^2x^2}d\sqrt{c+dx}$$
$$\Big\downarrow \text{5812}$$
$$2d\left(-\frac{1}{2}b\int -\frac{\sinh\left(a+b\sqrt{c+dx}\right)}{dx}d\sqrt{c+dx}-\frac{\cosh\left(a+b\sqrt{c+dx}\right)}{2dx}\right)$$

$$\Big\downarrow \text{5803}$$

$$2d\left(-\frac{1}{2}b\int\left(\frac{\sinh\left(a+b\sqrt{c+dx}\right)}{2\sqrt{c}\left(-c+\sqrt{c}-dx\right)}+\frac{\sinh\left(a+b\sqrt{c+dx}\right)}{2\sqrt{c}\left(\sqrt{c}+\sqrt{c+dx}\right)}\right)d\sqrt{c+dx}-\frac{\cosh\left(a+b\sqrt{c+dx}\right)}{2dx}\right)$$

$$\Big\downarrow \text{2009}$$

$$2d\left(-\frac{1}{2}b\left(\frac{\sinh\left(a-b\sqrt{c}\right)\text{Chi}\left(\sqrt{c}b+\sqrt{c+dx}b\right)}{2\sqrt{c}}-\frac{\sinh\left(a+b\sqrt{c}\right)\text{Chi}\left(b\sqrt{c}-b\sqrt{c+dx}\right)}{2\sqrt{c}}+\frac{\cosh\left(a+b\sqrt{c}\right)\text{Shi}(b\cdot}{2\sqrt{c}}\right.\right.$$

input
```
Int[Cosh[a + b*Sqrt[c + d*x]]/x^2,x]
```

output
```
2*d*(-1/2*Cosh[a + b*Sqrt[c + d*x]]/(d*x) - (b*((CoshIntegral[b*Sqrt[c] +
b*Sqrt[c + d*x]]*Sinh[a - b*Sqrt[c]])/(2*Sqrt[c]) - (CoshIntegral[b*Sqrt[c
] - b*Sqrt[c + d*x]]*Sinh[a + b*Sqrt[c]])/(2*Sqrt[c]) + (Cosh[a + b*Sqrt[c
]]*SinhIntegral[b*Sqrt[c] - b*Sqrt[c + d*x]])/(2*Sqrt[c]) + (Cosh[a - b*Sq
rt[c]]*SinhIntegral[b*Sqrt[c] + b*Sqrt[c + d*x]])/(2*Sqrt[c])))/2)
```

3.63.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5803
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 5812
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x],
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1,
0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

rule 5888
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/Coefficient[u, x, 1]^(m + 1)   Subst[Int[(x - Coefficient[u, x
, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p
}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.63.4 Maple [F]

$$\int \frac{\cosh\left(a+b\sqrt{dx+c}\right)}{x^2}dx$$

input
```
int(cosh(a+b*(d*x+c)^(1/2))/x^2,x)
```

output
```
int(cosh(a+b*(d*x+c)^(1/2))/x^2,x)
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(142) = 284$.

Time = 0.26 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.74

$$\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx = -\frac{4\,c\cosh\left(\sqrt{dx+c}b+a\right)-\left(\sqrt{b^2c}dx\mathrm{Ei}\left(\sqrt{dx+c}b-\sqrt{b^2c}\right)-\sqrt{b^2c}dx\mathrm{Ei}\left(-\sqrt{dx+c}b+\sqrt{b^2c}\right)\right)\cosh\left(a\right.}{}$$

input
```
integrate(cosh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="fricas")
```

output
```
-1/4*(4*c*cosh(sqrt(d*x + c)*b + a) - (sqrt(b^2*c)*d*x*Ei(sqrt(d*x + c)*b
- sqrt(b^2*c)) - sqrt(b^2*c)*d*x*Ei(-sqrt(d*x + c)*b + sqrt(b^2*c)))*cosh(
a + sqrt(b^2*c)) + (sqrt(b^2*c)*d*x*Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) - sq
rt(b^2*c)*d*x*Ei(-sqrt(d*x + c)*b - sqrt(b^2*c)))*cosh(-a + sqrt(b^2*c)) -
 (sqrt(b^2*c)*d*x*Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) + sqrt(b^2*c)*d*x*Ei(-
sqrt(d*x + c)*b + sqrt(b^2*c)))*sinh(a + sqrt(b^2*c)) - (sqrt(b^2*c)*d*x*E
i(sqrt(d*x + c)*b + sqrt(b^2*c)) + sqrt(b^2*c)*d*x*Ei(-sqrt(d*x + c)*b - s
qrt(b^2*c)))*sinh(-a + sqrt(b^2*c)))/(c*x)
```

3.63.6 Sympy [F]

$$\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx=\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx$$

input
```
integrate(cosh(a+b*(d*x+c)**(1/2))/x**2,x)
```

output
```
Integral(cosh(a + b*sqrt(c + d*x))/x**2, x)
```

3.63.7 Maxima [F]

$$\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx=\int \frac{\cosh\left(\sqrt{dx+c}b+a\right)}{x^2}\,dx$$

input
```
integrate(cosh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="maxima")
```

output
```
integrate(cosh(sqrt(d*x + c)*b + a)/x^2, x)
```

3.63.8 Giac [F]

$$\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx=\int \frac{\cosh\left(\sqrt{dx+c}b+a\right)}{x^2}\,dx$$

input
```
integrate(cosh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="giac")
```

output
```
integrate(cosh(sqrt(d*x + c)*b + a)/x^2, x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh \left(a+b\sqrt{c+dx}\right)}{x^2}\,dx=\int \frac{\cosh\left(a+b\,\sqrt{c+d\,x}\right)}{x^2}\,dx$$

input
```
int(cosh(a + b*(c + d*x)^(1/2))/x^2,x)
```

output
```
int(cosh(a + b*(c + d*x)^(1/2))/x^2, x)
```

3.64 $\int x^2 \cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx$

3.64.1 Optimal result

Integrand size = 18, antiderivative size = 537

$$\int x^2\cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx=\frac{720c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3}-\frac{120960\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3}-\frac{6c^2\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}+\frac{360c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3}-\frac{20160(c+dx)\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3}+\frac{30c(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}-\frac{1008(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3}-\frac{24(c+dx)^{7/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}+\frac{120960\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3}+\frac{6c^2\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}-\frac{720c\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3}+\frac{60480(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3}+\frac{3c^2(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}-\frac{120c(c+dx)\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}+\frac{5040(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3}-\frac{6c(c+dx)^{5/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}+\frac{168(c+dx)^2\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}+\frac{3(c+dx)^{8/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$

output
```
720*c*cosh(a+b*(d*x+c)^(1/3))/b^6/d^3-120960*(d*x+c)^(1/3)*cosh(a+b*(d*x+c
)^(1/3))/b^8/d^3-6*c^2*(d*x+c)^(1/3)*cosh(a+b*(d*x+c)^(1/3))/b^2/d^3+360*c
*(d*x+c)^(2/3)*cosh(a+b*(d*x+c)^(1/3))/b^4/d^3-20160*(d*x+c)*cosh(a+b*(d*x
+c)^(1/3))/b^6/d^3+30*c*(d*x+c)^(4/3)*cosh(a+b*(d*x+c)^(1/3))/b^2/d^3-1008
*(d*x+c)^(5/3)*cosh(a+b*(d*x+c)^(1/3))/b^4/d^3-24*(d*x+c)^(7/3)*cosh(a+b*(
d*x+c)^(1/3))/b^2/d^3+120960*sinh(a+b*(d*x+c)^(1/3))/b^9/d^3+6*c^2*sinh(a+
b*(d*x+c)^(1/3))/b^3/d^3-720*c*(d*x+c)^(1/3)*sinh(a+b*(d*x+c)^(1/3))/b^5/d
^3+60480*(d*x+c)^(2/3)*sinh(a+b*(d*x+c)^(1/3))/b^7/d^3+3*c^2*(d*x+c)^(2/3)
*sinh(a+b*(d*x+c)^(1/3))/b/d^3-120*c*(d*x+c)*sinh(a+b*(d*x+c)^(1/3))/b^3/d
^3+5040*(d*x+c)^(4/3)*sinh(a+b*(d*x+c)^(1/3))/b^5/d^3-6*c*(d*x+c)^(5/3)*si
nh(a+b*(d*x+c)^(1/3))/b/d^3+168*(d*x+c)^2*sinh(a+b*(d*x+c)^(1/3))/b^3/d^3+
3*(d*x+c)^(8/3)*sinh(a+b*(d*x+c)^(1/3))/b/d^3
```

3.64.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.66

$$\int x^2 \cosh\left(a + b\sqrt[3]{c+dx}\right) dx$$
$$= \frac{e^{-a-b\sqrt[3]{c+dx}}\left(120960\left(-1+e^{2\left(a+b\sqrt[3]{c+dx}\right)}\right) - 120960b\left(1+e^{2\left(a+b\sqrt[3]{c+dx}\right)}\right)\sqrt[3]{c+dx} + 60480b^2\left(-1+e^{2\left(a\right.}\right.\right.}{}$$

input
```
Integrate[x^2*Cosh[a + b*(c + d*x)^(1/3)],x]
```

output
```
(E^(-a - b*(c + d*x)^(1/3))*(120960*(-1 + E^(2*(a + b*(c + d*x)^(1/3)))) -
 120960*b*(1 + E^(2*(a + b*(c + d*x)^(1/3))))*(c + d*x)^(1/3) + 60480*b^2*
(-1 + E^(2*(a + b*(c + d*x)^(1/3))))*(c + d*x)^(2/3) + 3*b^8*d^2*(-1 + E^(
2*(a + b*(c + d*x)^(1/3))))*x^2*(c + d*x)^(2/3) - 6*b^7*d*(1 + E^(2*(a + b
*(c + d*x)^(1/3))))*x*(c + d*x)^(1/3)*(3*c + 4*d*x) + 720*b^4*(-1 + E^(2*(
a + b*(c + d*x)^(1/3))))*(c + d*x)^(1/3)*(6*c + 7*d*x) - 72*b^5*(1 + E^(2*
(a + b*(c + d*x)^(1/3))))*(c + d*x)^(2/3)*(9*c + 14*d*x) - 720*b^3*(1 + E^
(2*(a + b*(c + d*x)^(1/3))))*(27*c + 28*d*x) + 6*b^6*(-1 + E^(2*(a + b*(c
+ d*x)^(1/3))))*(9*c^2 + 36*c*d*x + 28*d^2*x^2)))/(2*b^9*d^3)
```

3.64.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 486, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5888, 7267, 2027, 5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cosh\left(a+b\sqrt[3]{c+dx}\right)dx$$

$$\Big\downarrow \text{5888}$$

$$\frac{\int d^2x^2\cosh\left(a+b\sqrt[3]{c+dx}\right)d(c+dx)}{d^3}$$

$$\Big\downarrow \text{7267}$$

$$\frac{3\int\left(c\sqrt[3]{c+dx}-(c+dx)^{4/3}\right)^2\cosh\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{d^3}$$

$$\Big\downarrow \text{2027}$$

$$\frac{3\int d^2x^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{d^3}$$

$$\Big\downarrow \text{5810}$$

$$\frac{3\int\left(\cosh\left(a+b\sqrt[3]{c+dx}\right)(c+dx)^{8/3}-2c\cosh\left(a+b\sqrt[3]{c+dx}\right)(c+dx)^{5/3}+c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)(c+dx)^{2/}\right.}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{40320\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^9}-\frac{40320\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^8}+\frac{20160(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^7}-\frac{6720(c+dx)\cosh\left(\right.}{b^6}\right.$$

input
```
Int[x^2*Cosh[a + b*(c + d*x)^(1/3)],x]
```

output
```
(3*((240*c*Cosh[a + b*(c + d*x)^(1/3)])/b^6 - (40320*(c + d*x)^(1/3)*Cosh[
a + b*(c + d*x)^(1/3)])/b^8 - (2*c^2*(c + d*x)^(1/3)*Cosh[a + b*(c + d*x)^
(1/3)])/b^2 + (120*c*(c + d*x)^(2/3)*Cosh[a + b*(c + d*x)^(1/3)])/b^4 - (6
720*(c + d*x)*Cosh[a + b*(c + d*x)^(1/3)])/b^6 + (10*c*(c + d*x)^(4/3)*Cos
h[a + b*(c + d*x)^(1/3)])/b^2 - (336*(c + d*x)^(5/3)*Cosh[a + b*(c + d*x)^
(1/3)])/b^4 - (8*(c + d*x)^(7/3)*Cosh[a + b*(c + d*x)^(1/3)])/b^2 + (40320
*Sinh[a + b*(c + d*x)^(1/3)])/b^9 + (2*c^2*Sinh[a + b*(c + d*x)^(1/3)])/b^
3 - (240*c*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x)^(1/3)])/b^5 + (20160*(c +
d*x)^(2/3)*Sinh[a + b*(c + d*x)^(1/3)])/b^7 + (c^2*(c + d*x)^(2/3)*Sinh[a
+ b*(c + d*x)^(1/3)])/b - (40*c*(c + d*x)*Sinh[a + b*(c + d*x)^(1/3)])/b^3
 + (1680*(c + d*x)^(4/3)*Sinh[a + b*(c + d*x)^(1/3)])/b^5 - (2*c*(c + d*x)
^(5/3)*Sinh[a + b*(c + d*x)^(1/3)])/b + (56*(c + d*x)^2*Sinh[a + b*(c + d*
x)^(1/3)])/b^3 + ((c + d*x)^(8/3)*Sinh[a + b*(c + d*x)^(1/3)])/b))/d^3
```

3.64.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 5888
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/Coefficient[u, x, 1]^(m + 1)   Subst[Int[(x - Coefficient[u, x
, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p
}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.64.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1814 vs. $2(477) = 954$.

Time = 0.21 (sec) , antiderivative size = 1815, normalized size of antiderivative = 3.38

method	result	size
derivativedivides	Expression too large to display	1815
default	Expression too large to display	1815
parts	Expression too large to display	2938

input
```
int(x^2*cosh(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
3/d^3/b^3*(1/b^6*((a+b*(d*x+c)^(1/3))^8*sinh(a+b*(d*x+c)^(1/3))-8*(a+b*(d*
x+c)^(1/3))^7*cosh(a+b*(d*x+c)^(1/3))+56*(a+b*(d*x+c)^(1/3))^6*sinh(a+b*(d
*x+c)^(1/3))-336*(a+b*(d*x+c)^(1/3))^5*cosh(a+b*(d*x+c)^(1/3))+1680*(a+b*(
d*x+c)^(1/3))^4*sinh(a+b*(d*x+c)^(1/3))-6720*(a+b*(d*x+c)^(1/3))^3*cosh(a+
b*(d*x+c)^(1/3))+20160*(a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))-40320
*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))+40320*sinh(a+b*(d*x+c)^(1/3))
)+1/b^6*a^8*sinh(a+b*(d*x+c)^(1/3))+c^2*a^2*sinh(a+b*(d*x+c)^(1/3))+c^2*((
a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))-2*(a+b*(d*x+c)^(1/3))*cosh(a+
b*(d*x+c)^(1/3))+2*sinh(a+b*(d*x+c)^(1/3)))-56/b^6*a^5*((a+b*(d*x+c)^(1/3)
)^3*sinh(a+b*(d*x+c)^(1/3))-3*(a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3)
)+6*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))-6*cosh(a+b*(d*x+c)^(1/3)))
+2/b^3*a^5*c*sinh(a+b*(d*x+c)^(1/3))+70/b^6*a^4*((a+b*(d*x+c)^(1/3))^4*sin
h(a+b*(d*x+c)^(1/3))-4*(a+b*(d*x+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))+12*(a
+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*cosh(a+
b*(d*x+c)^(1/3))+24*sinh(a+b*(d*x+c)^(1/3)))-56/b^6*a^3*((a+b*(d*x+c)^(1/3
))^5*sinh(a+b*(d*x+c)^(1/3))-5*(a+b*(d*x+c)^(1/3))^4*cosh(a+b*(d*x+c)^(1/3
))+20*(a+b*(d*x+c)^(1/3))^3*sinh(a+b*(d*x+c)^(1/3))-60*(a+b*(d*x+c)^(1/3))
^2*cosh(a+b*(d*x+c)^(1/3))+120*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))
-120*cosh(a+b*(d*x+c)^(1/3)))-2/b^3*c*((a+b*(d*x+c)^(1/3))^5*sinh(a+b*(d*x
+c)^(1/3))-5*(a+b*(d*x+c)^(1/3))^4*cosh(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x...
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.34

$$\int x^2\cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx = -\frac{3\left(2\left(3360\,b^3dx+3240\,b^3c+12\,(14\,b^5dx+9\,b^5c)(dx+c)^{\frac{2}{3}}+(4\,b^7d^2x^2+3\,b^7cdx+20160\,b)(dx+c)^{\frac{1}{3}}\right)\right.}{}$$

input
```
integrate(x^2*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

output
```
-3*(2*(3360*b^3*d*x + 3240*b^3*c + 12*(14*b^5*d*x + 9*b^5*c)*(d*x + c)^(2/
3) + (4*b^7*d^2*x^2 + 3*b^7*c*d*x + 20160*b)*(d*x + c)^(1/3))*cosh((d*x +
c)^(1/3)*b + a) - (56*b^6*d^2*x^2 + 72*b^6*c*d*x + 18*b^6*c^2 + (b^8*d^2*x
^2 + 20160*b^2)*(d*x + c)^(2/3) + 240*(7*b^4*d*x + 6*b^4*c)*(d*x + c)^(1/3
) + 40320)*sinh((d*x + c)^(1/3)*b + a))/(b^9*d^3)
```

3.64.6 Sympy [F]

$$\int x^2\cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx = \int x^2\cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

input
```
integrate(x**2*cosh(a+b*(d*x+c)**(1/3)),x)
```

output
```
Integral(x**2*cosh(a + b*(c + d*x)**(1/3)), x)
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.20

$$\int x^2\cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

$$= \frac{2\,d^3x^3\cosh\left((dx+c)^{\frac{1}{3}}b+a\right)+\left(\frac{c^3e^{\left((dx+c)^{\frac{1}{3}}b+a\right)}}{b}+\frac{c^3e^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{b}-\frac{3\left((dx+c)b^3e^a-3\,(dx+c)^{\frac{2}{3}}b^2e^a+6\,(dx+c)^{\frac{1}{3}}be^a-6\right.}{b^4}\right.}{}$$

input
```
integrate(x^2*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

output
```
1/6*(2*d^3*x^3*cosh((d*x + c)^(1/3)*b + a) + (c^3*e^((d*x + c)^(1/3)*b + a
)/b + c^3*e^(-(d*x + c)^(1/3)*b - a)/b - 3*((d*x + c)*b^3*e^a - 3*(d*x + c
)^(2/3)*b^2*e^a + 6*(d*x + c)^(1/3)*b*e^a - 6*e^a)*c^2*e^((d*x + c)^(1/3)*
b)/b^4 - 3*((d*x + c)*b^3 + 3*(d*x + c)^(2/3)*b^2 + 6*(d*x + c)^(1/3)*b +
6)*c^2*e^(-(d*x + c)^(1/3)*b - a)/b^4 + 3*((d*x + c)^2*b^6*e^a - 6*(d*x +
c)^(5/3)*b^5*e^a + 30*(d*x + c)^(4/3)*b^4*e^a - 120*(d*x + c)*b^3*e^a + 36
0*(d*x + c)^(2/3)*b^2*e^a - 720*(d*x + c)^(1/3)*b*e^a + 720*e^a)*c*e^((d*x
 + c)^(1/3)*b)/b^7 + 3*((d*x + c)^2*b^6 + 6*(d*x + c)^(5/3)*b^5 + 30*(d*x
+ c)^(4/3)*b^4 + 120*(d*x + c)*b^3 + 360*(d*x + c)^(2/3)*b^2 + 720*(d*x +
c)^(1/3)*b + 720)*c*e^(-(d*x + c)^(1/3)*b - a)/b^7 - ((d*x + c)^3*b^9*e^a
- 9*(d*x + c)^(8/3)*b^8*e^a + 72*(d*x + c)^(7/3)*b^7*e^a - 504*(d*x + c)^2
*b^6*e^a + 3024*(d*x + c)^(5/3)*b^5*e^a - 15120*(d*x + c)^(4/3)*b^4*e^a +
60480*(d*x + c)*b^3*e^a - 181440*(d*x + c)^(2/3)*b^2*e^a + 362880*(d*x + c
)^(1/3)*b*e^a - 362880*e^a)*e^((d*x + c)^(1/3)*b)/b^10 - ((d*x + c)^3*b^9
+ 9*(d*x + c)^(8/3)*b^8 + 72*(d*x + c)^(7/3)*b^7 + 504*(d*x + c)^2*b^6 + 3
024*(d*x + c)^(5/3)*b^5 + 15120*(d*x + c)^(4/3)*b^4 + 60480*(d*x + c)*b^3
+ 181440*(d*x + c)^(2/3)*b^2 + 362880*(d*x + c)^(1/3)*b + 362880)*e^(-(d*x
 + c)^(1/3)*b - a)/b^10)*b)/d^3
```

3.64.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2163 vs. $2(477) = 954$.

Time = 0.36 (sec) , antiderivative size = 2163, normalized size of antiderivative = 4.03

$$\int x^2 \cosh\left(a + b\sqrt[3]{c + dx}\right) dx = \text{Too large to display}$$

input
```
integrate(x^2*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

output
```
3/2*((((d*x + c)^(1/3)*b + a)^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)*a*b^6*
c^2 + a^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)^5*b^3*c + 10*((d*x + c)^(1/3
)*b + a)^4*a*b^3*c - 20*((d*x + c)^(1/3)*b + a)^3*a^2*b^3*c + 20*((d*x + c
)^(1/3)*b + a)^2*a^3*b^3*c - 10*((d*x + c)^(1/3)*b + a)*a^4*b^3*c + 2*a^5*
b^3*c - 2*((d*x + c)^(1/3)*b + a)*b^6*c^2 + 2*a*b^6*c^2 + ((d*x + c)^(1/3)
*b + a)^8 - 8*((d*x + c)^(1/3)*b + a)^7*a + 28*((d*x + c)^(1/3)*b + a)^6*a
^2 - 56*((d*x + c)^(1/3)*b + a)^5*a^3 + 70*((d*x + c)^(1/3)*b + a)^4*a^4 -
 56*((d*x + c)^(1/3)*b + a)^3*a^5 + 28*((d*x + c)^(1/3)*b + a)^2*a^6 - 8*(
(d*x + c)^(1/3)*b + a)*a^7 + a^8 + 10*((d*x + c)^(1/3)*b + a)^4*b^3*c - 40
*((d*x + c)^(1/3)*b + a)^3*a*b^3*c + 60*((d*x + c)^(1/3)*b + a)^2*a^2*b^3*
c - 40*((d*x + c)^(1/3)*b + a)*a^3*b^3*c + 10*a^4*b^3*c + 2*b^6*c^2 - 8*((
d*x + c)^(1/3)*b + a)^7 + 56*((d*x + c)^(1/3)*b + a)^6*a - 168*((d*x + c)^
(1/3)*b + a)^5*a^2 + 280*((d*x + c)^(1/3)*b + a)^4*a^3 - 280*((d*x + c)^(1
/3)*b + a)^3*a^4 + 168*((d*x + c)^(1/3)*b + a)^2*a^5 - 56*((d*x + c)^(1/3)
*b + a)*a^6 + 8*a^7 - 40*((d*x + c)^(1/3)*b + a)^3*b^3*c + 120*((d*x + c)^
(1/3)*b + a)^2*a*b^3*c - 120*((d*x + c)^(1/3)*b + a)*a^2*b^3*c + 40*a^3*b^
3*c + 56*((d*x + c)^(1/3)*b + a)^6 - 336*((d*x + c)^(1/3)*b + a)^5*a + 840
*((d*x + c)^(1/3)*b + a)^4*a^2 - 1120*((d*x + c)^(1/3)*b + a)^3*a^3 + 840*
((d*x + c)^(1/3)*b + a)^2*a^4 - 336*((d*x + c)^(1/3)*b + a)*a^5 + 56*a^6 +
 120*((d*x + c)^(1/3)*b + a)^2*b^3*c - 240*((d*x + c)^(1/3)*b + a)*a*b^...
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh\left(a + b\sqrt[3]{c + dx}\right) dx = \int x^2 \cosh\left(a + b\left(c + d\,x\right)^{1/3}\right) dx$$

input
```
int(x^2*cosh(a + b*(c + d*x)^(1/3)),x)
```

output
```
int(x^2*cosh(a + b*(c + d*x)^(1/3)), x)
```

3.65 $\int x\cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx$

3.65.1 Optimal result

Integrand size = 16, antiderivative size = 261

$$\begin{aligned}\int x\cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx =& -\frac{360\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2} \\ &+ \frac{6c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} \\ &- \frac{180(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2} \\ &- \frac{15(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} \\ &- \frac{6c\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} \\ &+ \frac{360\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} \\ &- \frac{3c(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} \\ &+ \frac{60(c+dx)\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} \\ &+ \frac{3(c+dx)^{5/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}\end{aligned}$$

output
```
-360*cosh(a+b*(d*x+c)^(1/3))/b^6/d^2+6*c*(d*x+c)^(1/3)*cosh(a+b*(d*x+c)^(1
/3))/b^2/d^2-180*(d*x+c)^(2/3)*cosh(a+b*(d*x+c)^(1/3))/b^4/d^2-15*(d*x+c)^
(4/3)*cosh(a+b*(d*x+c)^(1/3))/b^2/d^2-6*c*sinh(a+b*(d*x+c)^(1/3))/b^3/d^2+
360*(d*x+c)^(1/3)*sinh(a+b*(d*x+c)^(1/3))/b^5/d^2-3*c*(d*x+c)^(2/3)*sinh(a
+b*(d*x+c)^(1/3))/b/d^2+60*(d*x+c)*sinh(a+b*(d*x+c)^(1/3))/b^3/d^2+3*(d*x+
c)^(5/3)*sinh(a+b*(d*x+c)^(1/3))/b/d^2
```

3.65.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.45

$$\int x\cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{-3\left(120+60b^2(c+dx)^{2/3}+b^4\sqrt[3]{c+dx}(3c+5dx)\right)\cosh\left(a+b\sqrt[3]{c+dx}\right)+3b\left(120\sqrt[3]{c+dx}+b^4dx(c+\ldots\right.}{b^6d^2}$$

input
```
Integrate[x*Cosh[a + b*(c + d*x)^(1/3)],x]
```

output
```
(-3*(120 + 60*b^2*(c + d*x)^(2/3) + b^4*(c + d*x)^(1/3)*(3*c + 5*d*x))*Cos
h[a + b*(c + d*x)^(1/3)] + 3*b*(120*(c + d*x)^(1/3) + b^4*d*x*(c + d*x)^(2
/3) + 2*b^2*(9*c + 10*d*x))*Sinh[a + b*(c + d*x)^(1/3)])/(b^6*d^2)
```

3.65.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5888, 25, 7267, 5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

$$\Big\downarrow \ 5888$$

$$\frac{\int dx\cosh\left(a+b\sqrt[3]{c+dx}\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -dx \cosh\left(a+b\sqrt[3]{c+dx}\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \text{7267}$$

$$-\frac{3\int -dx(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{d^2}$$

$$\Big\downarrow \text{5810}$$

$$-\frac{3\int \left(c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)-(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)\right)d\sqrt[3]{c+dx}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\left(\frac{120\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^6}-\frac{120\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^5}+\frac{60(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^4}-\frac{20(c+dx)\sinh\left(a+b\sqrt[3]{c\right.}}{b^3}}{}$$

input
```
Int[x*Cosh[a + b*(c + d*x)^(1/3)],x]
```

output
```
(-3*((120*Cosh[a + b*(c + d*x)^(1/3)])/b^6 - (2*c*(c + d*x)^(1/3)*Cosh[a +
 b*(c + d*x)^(1/3)])/b^2 + (60*(c + d*x)^(2/3)*Cosh[a + b*(c + d*x)^(1/3)]
)/b^4 + (5*(c + d*x)^(4/3)*Cosh[a + b*(c + d*x)^(1/3)])/b^2 + (2*c*Sinh[a
+ b*(c + d*x)^(1/3)])/b^3 - (120*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x)^(1/3
)])/b^5 + (c*(c + d*x)^(2/3)*Sinh[a + b*(c + d*x)^(1/3)])/b - (20*(c + d*x
)*Sinh[a + b*(c + d*x)^(1/3)])/b^3 - ((c + d*x)^(5/3)*Sinh[a + b*(c + d*x)
^(1/3)])/b))/d^2
```

3.65.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 5888
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/Coefficient[u, x, 1]^(m + 1)   Subst[Int[(x - Coefficient[u, x
, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p
}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.65.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(231) = 462$.

Time = 0.19 (sec) , antiderivative size = 659, normalized size of antiderivative = 2.52

method	result
derivativedivides	$\frac{-\frac{3\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)a^5}{b^3}+\frac{15a^4\left(\left(a+b(dx+c)^{\frac{1}{3}}\right)\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)-\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)}{b^3}-\frac{30a^3\left(\left(a+b(dx+c)^{\frac{1}{3}}\right)^2\sinh\left(a\cdots\right.\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{3\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)a^5}{b^3}+\frac{15a^4\left(\left(a+b(dx+c)^{\frac{1}{3}}\right)\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)-\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)}{b^3}-\frac{30a^3\left(\left(a+b(dx+c)^{\frac{1}{3}}\right)^2\sinh\left(a\cdots\right.\right.}{\cdots}}{\cdots}$
parts	Expression too large to display

input
```
int(x*cosh(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
3/d^2/b^3*(-sinh(a+b*(d*x+c)^(1/3))*a^5/b^3+5/b^3*a^4*((a+b*(d*x+c)^(1/3))
*sinh(a+b*(d*x+c)^(1/3))-cosh(a+b*(d*x+c)^(1/3)))-10/b^3*a^3*((a+b*(d*x+c)
^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))-2*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(
1/3))+2*sinh(a+b*(d*x+c)^(1/3)))+10/b^3*a^2*((a+b*(d*x+c)^(1/3))^3*sinh(a+
b*(d*x+c)^(1/3))-3*(a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))+6*(a+b*(d
*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))-6*cosh(a+b*(d*x+c)^(1/3)))-5/b^3*a*((
a+b*(d*x+c)^(1/3))^4*sinh(a+b*(d*x+c)^(1/3))-4*(a+b*(d*x+c)^(1/3))^3*cosh(
a+b*(d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))-24*(a+
b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))+24*sinh(a+b*(d*x+c)^(1/3)))+1/b^3
*((a+b*(d*x+c)^(1/3))^5*sinh(a+b*(d*x+c)^(1/3))-5*(a+b*(d*x+c)^(1/3))^4*co
sh(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*sinh(a+b*(d*x+c)^(1/3))-60*
(a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))+120*(a+b*(d*x+c)^(1/3))*sinh
(a+b*(d*x+c)^(1/3))-120*cosh(a+b*(d*x+c)^(1/3)))-c*a^2*sinh(a+b*(d*x+c)^(1
/3))+2*c*a*((a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))-cosh(a+b*(d*x+c)^(
1/3)))-c*((a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))-2*(a+b*(d*x+c)^(1/
3))*cosh(a+b*(d*x+c)^(1/3))+2*sinh(a+b*(d*x+c)^(1/3))))
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.42

$$\int x\cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx = -\frac{3\left(\left(60\,(dx+c)^{\frac{2}{3}}b^2+(5\,b^4dx+3\,b^4c)(dx+c)^{\frac{1}{3}}+120\right)\cosh\left((dx+c)^{\frac{1}{3}}b+a\right)-\left((dx+c)^{\frac{2}{3}}b^5dx+20\,b\right.}{b^6d^2}$$

input
```
integrate(x*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

output
```
-3*((60*(d*x + c)^(2/3)*b^2 + (5*b^4*d*x + 3*b^4*c)*(d*x + c)^(1/3) + 120)
*cosh((d*x + c)^(1/3)*b + a) - ((d*x + c)^(2/3)*b^5*d*x + 20*b^3*d*x + 18*
b^3*c + 120*(d*x + c)^(1/3)*b)*sinh((d*x + c)^(1/3)*b + a))/(b^6*d^2)
```

3.65.6 Sympy [F]

$$\int x\cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx=\int x\cosh\left(a+b\sqrt[3]{c+dx}\right)dx$$

input
```
integrate(x*cosh(a+b*(d*x+c)**(1/3)),x)
```

output
```
Integral(x*cosh(a + b*(c + d*x)**(1/3)), x)
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.41

$$\int x\cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx$$
$$=\frac{2\,d^2x^2\cosh\left((dx+c)^{\frac{1}{3}}b+a\right)-\left(\frac{c^2e^{\left((dx+c)^{\frac{1}{3}}b+a\right)}}{b}+\frac{c^2e^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{b}-\frac{2\left((dx+c)b^3e^a-3\,(dx+c)^{\frac{2}{3}}b^2e^a+6\,(dx+c)^{\frac{1}{3}}be^a-6\right.}{b^4}\right.}{}$$

input
```
integrate(x*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

output
```
1/4*(2*d^2*x^2*cosh((d*x + c)^(1/3)*b + a) - (c^2*e^((d*x + c)^(1/3)*b + a
)/b + c^2*e^(-(d*x + c)^(1/3)*b - a)/b - 2*((d*x + c)*b^3*e^a - 3*(d*x + c
)^(2/3)*b^2*e^a + 6*(d*x + c)^(1/3)*b*e^a - 6*e^a)*c*e^((d*x + c)^(1/3)*b)
/b^4 - 2*((d*x + c)*b^3 + 3*(d*x + c)^(2/3)*b^2 + 6*(d*x + c)^(1/3)*b + 6)
*c*e^(-(d*x + c)^(1/3)*b - a)/b^4 + ((d*x + c)^2*b^6*e^a - 6*(d*x + c)^(5/
3)*b^5*e^a + 30*(d*x + c)^(4/3)*b^4*e^a - 120*(d*x + c)*b^3*e^a + 360*(d*x
 + c)^(2/3)*b^2*e^a - 720*(d*x + c)^(1/3)*b*e^a + 720*e^a)*e^((d*x + c)^(1
/3)*b)/b^7 + ((d*x + c)^2*b^6 + 6*(d*x + c)^(5/3)*b^5 + 30*(d*x + c)^(4/3)
*b^4 + 120*(d*x + c)*b^3 + 360*(d*x + c)^(2/3)*b^2 + 720*(d*x + c)^(1/3)*b
 + 720)*e^(-(d*x + c)^(1/3)*b - a)/b^7)*b)/d^2
```

3.65.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(231) = 462$.

Time = 0.28 (sec) , antiderivative size = 707, normalized size of antiderivative = 2.71

$$\int x\cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx =$$

$$-\frac{3\left(\frac{\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2 b^3c-2\left((dx+c)^{\frac{1}{3}}b+a\right)ab^3c+a^2b^3c-\left((dx+c)^{\frac{1}{3}}b+a\right)^5+5\left((dx+c)^{\frac{1}{3}}b+a\right)^4a-10\left((dx+c)^{\frac{1}{3}}b+a\right)^3a^2+10\left((dx+c)^{\frac{1}{3}}b+\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate(x*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

output
```
-3/2*((((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x + c)^(1/3)*b + a)*a*b^3*c
 + a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x + c)^(1/3)*b + a)^4*a -
 10*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(1/3)*b + a)^2*a^3 - 5*(
(d*x + c)^(1/3)*b + a)*a^4 + a^5 - 2*((d*x + c)^(1/3)*b + a)*b^3*c + 2*a*b
^3*c + 5*((d*x + c)^(1/3)*b + a)^4 - 20*((d*x + c)^(1/3)*b + a)^3*a + 30*(
(d*x + c)^(1/3)*b + a)^2*a^2 - 20*((d*x + c)^(1/3)*b + a)*a^3 + 5*a^4 + 2*
b^3*c - 20*((d*x + c)^(1/3)*b + a)^3 + 60*((d*x + c)^(1/3)*b + a)^2*a - 60
*((d*x + c)^(1/3)*b + a)*a^2 + 20*a^3 + 60*((d*x + c)^(1/3)*b + a)^2 - 120
*((d*x + c)^(1/3)*b + a)*a + 60*a^2 - 120*(d*x + c)^(1/3)*b + 120)*e^((d*x
 + c)^(1/3)*b + a)/(b^5*d) - ((((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x +
c)^(1/3)*b + a)*a*b^3*c + a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x
+ c)^(1/3)*b + a)^4*a - 10*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(
1/3)*b + a)^2*a^3 - 5*((d*x + c)^(1/3)*b + a)*a^4 + a^5 + 2*((d*x + c)^(1/
3)*b + a)*b^3*c - 2*a*b^3*c - 5*((d*x + c)^(1/3)*b + a)^4 + 20*((d*x + c)^
(1/3)*b + a)^3*a - 30*((d*x + c)^(1/3)*b + a)^2*a^2 + 20*((d*x + c)^(1/3)*
b + a)*a^3 - 5*a^4 + 2*b^3*c - 20*((d*x + c)^(1/3)*b + a)^3 + 60*((d*x + c
)^(1/3)*b + a)^2*a - 60*((d*x + c)^(1/3)*b + a)*a^2 + 20*a^3 - 60*((d*x +
c)^(1/3)*b + a)^2 + 120*((d*x + c)^(1/3)*b + a)*a - 60*a^2 - 120*(d*x + c)
^(1/3)*b - 120)*e^(-(d*x + c)^(1/3)*b - a)/(b^5*d))/(b*d)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int x \cosh \left(a+b\sqrt[3]{c+dx}\right) \, dx=\int x\cosh\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right) \,dx$$

input
```
int(x*cosh(a + b*(c + d*x)^(1/3)),x)
```

output
```
int(x*cosh(a + b*(c + d*x)^(1/3)), x)
```

3.66 $\int \cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx$

3.66.1 Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx=-\frac{6\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{6\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}+\frac{3(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

output
```
-6*(d*x+c)^(1/3)*cosh(a+b*(d*x+c)^(1/3))/b^2/d+6*sinh(a+b*(d*x+c)^(1/3))/b
^3/d+3*(d*x+c)^(2/3)*sinh(a+b*(d*x+c)^(1/3))/b/d
```

3.66.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx=\frac{-6b\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)+3\left(2+b^2(c+dx)^{2/3}\right)\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}$$

input
```
Integrate[Cosh[a + b*(c + d*x)^(1/3)],x]
```

output
```
(-6*b*(c + d*x)^(1/3)*Cosh[a + b*(c + d*x)^(1/3)] + 3*(2 + b^2*(c + d*x)^(
2/3))*Sinh[a + b*(c + d*x)^(1/3)])/(b^3*d)
```

3.66.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5834, 5828, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

$$\Big\downarrow \text{5834}$$

$$\frac{\int \cosh\left(a+b\sqrt[3]{c+dx}\right)d(c+dx)}{d}$$

$$\Big\downarrow \text{5828}$$

$$\frac{3\int (c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int (c+dx)^{2/3}\sin\left(ia+ib\sqrt[3]{c+dx}+\frac{\pi}{2}\right)d\sqrt[3]{c+dx}}{d}$$

$$\Big\downarrow \text{3777}$$

$$\frac{3\left(\frac{(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{2i\int -i\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{3\left(\frac{(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{2\int \sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{2\int -i\sqrt[3]{c+dx}\sin\left(ia+ib\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{3\left(\frac{(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b}+\frac{2i\int\sqrt[3]{c+dx}\sin\left(ia+ib\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right)}{d}$$

$$\Big\downarrow \text{3777}$$

$$\frac{3\left(\frac{(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b}+\frac{2i\left(\frac{i\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{i\int\cosh\left(a+b\sqrt[3]{c+dx}\right)d\sqrt[3]{c+dx}}{b}\right)}{b}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b}+\frac{2i\left(\frac{i\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{i\int\sin\left(ia+ib\sqrt[3]{c+dx}+\frac{\pi}{2}\right)d\sqrt[3]{c+dx}}{b}\right)}{b}\right)}{d}$$

$$\Big\downarrow \text{3117}$$

$$\frac{3\left(\frac{(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b}+\frac{2i\left(\frac{i\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b}-\frac{i\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2}\right)}{b}\right)}{d}$$

input
```
Int[Cosh[a + b*(c + d*x)^(1/3)],x]
```

output
```
(3*(((c + d*x)^(2/3)*Sinh[a + b*(c + d*x)^(1/3)])/b + ((2*I)*((I*(c + d*x)
^(1/3)*Cosh[a + b*(c + d*x)^(1/3)])/b - (I*Sinh[a + b*(c + d*x)^(1/3)])/b^
2))/b))/d
```

3.66. $\int \cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx$

3.66.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5828
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.), x_Symbol] :> Modul
e[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Cosh[c + d*x^(k
*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d}, x] && FractionQ[n] &&
IntegerQ[p]
```

rule 5834
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.), x_Symbol] :> Simp[
1/Coefficient[u, x, 1]   Subst[Int[(a + b*Cosh[c + d*x^n])^p, x], x, u], x]
 /; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

3.66.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

method	result
derivativedivides	$\frac{3a^2 \sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)-6a\left(\left(a+b(dx+c)^{\frac{1}{3}}\right)\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)-\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)+3\left(a+b(dx+c)^{\frac{1}{3}}\right)^2\sinh\left(a+\right.}{d\,b^3}$
default	$\frac{3a^2 \sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)-6a\left(\left(a+b(dx+c)^{\frac{1}{3}}\right)\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)-\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)+3\left(a+b(dx+c)^{\frac{1}{3}}\right)^2\sinh\left(a+\right.}{d\,b^3}$

input
```
int(cosh(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
3/d/b^3*(a^2*sinh(a+b*(d*x+c)^(1/3))-2*a*((a+b*(d*x+c)^(1/3))*sinh(a+b*(d*
x+c)^(1/3))-cosh(a+b*(d*x+c)^(1/3)))+(a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c
)^(1/3))-2*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))+2*sinh(a+b*(d*x+c)^
(1/3)))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

$$\int \cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx = -\frac{3\left(2\,(dx+c)^{\frac{1}{3}}b\cosh\left((dx+c)^{\frac{1}{3}}b+a\right)-\left((dx+c)^{\frac{2}{3}}b^2+2\right)\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^3d}$$

input
```
integrate(cosh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

output
```
-3*(2*(d*x + c)^(1/3)*b*cosh((d*x + c)^(1/3)*b + a) - ((d*x + c)^(2/3)*b^2
 + 2)*sinh((d*x + c)^(1/3)*b + a))/(b^3*d)
```

3.66.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx = \begin{cases} x\cosh(a) & \text{for } b=0 \wedge (b=0 \vee d \\ x\cosh\left(a+b\sqrt[3]{c}\right) & \text{for } d=0 \\ \frac{3(c+dx)^{\frac{2}{3}}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd}-\frac{6\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{6\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(a+b*(d*x+c)**(1/3)),x)
```

output
```
Piecewise((x*cosh(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*cosh(a + b*c**
(1/3)), Eq(d, 0)), (3*(c + d*x)**(2/3)*sinh(a + b*(c + d*x)**(1/3))/(b*d)
- 6*(c + d*x)**(1/3)*cosh(a + b*(c + d*x)**(1/3))/(b**2*d) + 6*sinh(a + b*
(c + d*x)**(1/3))/(b**3*d), True))
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.60

$$\int \cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx = -\frac{b\left(\frac{\left((dx+c)b^3e^a-3\,(dx+c)^{\frac{2}{3}}b^2e^a+6\,(dx+c)^{\frac{1}{3}}be^a-6\,e^a\right)e^{\left((dx+c)^{\frac{1}{3}}b\right)}}{b^4}+\frac{\left((dx+c)b^3+3\,(dx+c)^{\frac{2}{3}}b^2+6\,(dx+c)^{\frac{1}{3}}b+6\right)e^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{b^4}\right) -}{2\,d}$$

input
```
integrate(cosh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

output
```
-1/2*(b*(((d*x + c)*b^3*e^a - 3*(d*x + c)^(2/3)*b^2*e^a + 6*(d*x + c)^(1/3
)*b*e^a - 6*e^a)*e^((d*x + c)^(1/3)*b)/b^4 + ((d*x + c)*b^3 + 3*(d*x + c)^
(2/3)*b^2 + 6*(d*x + c)^(1/3)*b + 6)*e^(-(d*x + c)^(1/3)*b - a)/b^4) - 2*(
d*x + c)*cosh((d*x + c)^(1/3)*b + a))/d
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.51

$$\int \cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{3\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2-2\left((dx+c)^{\frac{1}{3}}b+a\right)a+a^2-2\,(dx+c)^{\frac{1}{3}}b+2\right)e^{\left((dx+c)^{\frac{1}{3}}b+a\right)}}{2\,b^3d} - \frac{3\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2-2\left((dx+c)^{\frac{1}{3}}b+a\right)a+a^2+2\,(dx+c)^{\frac{1}{3}}b+2\right)e^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{2\,b^3d}$$

input
```
integrate(cosh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

output
```
3/2*(((d*x + c)^(1/3)*b + a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2*(d*
x + c)^(1/3)*b + 2)*e^((d*x + c)^(1/3)*b + a)/(b^3*d) - 3/2*(((d*x + c)^(1
/3)*b + a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 + 2*(d*x + c)^(1/3)*b + 2
)*e^(-(d*x + c)^(1/3)*b - a)/(b^3*d)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \cosh \left(a+b\sqrt[3]{c+dx}\right) \, dx = \frac{6\sinh\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)}{b^3\,d} - \frac{6\cosh\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,{\left(c+d\,x\right)}^{1/3}}{b^2\,d} + \frac{3\sinh\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,{\left(c+d\,x\right)}^{2/3}}{b\,d}$$

input
```
int(cosh(a + b*(c + d*x)^(1/3)),x)
```

output
```
(6*sinh(a + b*(c + d*x)^(1/3)))/(b^3*d) - (6*cosh(a + b*(c + d*x)^(1/3))*(
c + d*x)^(1/3))/(b^2*d) + (3*sinh(a + b*(c + d*x)^(1/3))*(c + d*x)^(2/3))/
(b*d)
```

3.67 $\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx$

3.67.1 Optimal result

Integrand size = 18, antiderivative size = 232

$$\begin{aligned}\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx &= \cosh\left(a+b\sqrt[3]{c}\right)\operatorname{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)\\
&+\cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\operatorname{Chi}\left(-b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)\\
&+\cosh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\operatorname{Chi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)\\
&-\sinh\left(a+b\sqrt[3]{c}\right)\operatorname{Shi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)\\
&-\sinh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\operatorname{Shi}\left(b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)+\sinh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\operatorname{Shi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)\end{aligned}$$

output
```
Chi(b*(c^(1/3)-(d*x+c)^(1/3)))*cosh(a+b*c^(1/3))+Chi(b*((-1)^(1/3)*c^(1/3)
+(d*x+c)^(1/3)))*cosh(a-(-1)^(1/3)*b*c^(1/3))+Chi(-b*((-1)^(2/3)*c^(1/3)-(
d*x+c)^(1/3)))*cosh(a+(-1)^(2/3)*b*c^(1/3))-Shi(b*(c^(1/3)-(d*x+c)^(1/3)))
*sinh(a+b*c^(1/3))+Shi(b*((-1)^(1/3)*c^(1/3)+(d*x+c)^(1/3)))*sinh(a-(-1)^(
1/3)*b*c^(1/3))-Shi(b*((-1)^(2/3)*c^(1/3)-(d*x+c)^(1/3)))*sinh(a+(-1)^(2/3
)*b*c^(1/3))
```

3.67.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx = \frac{1}{2}\Big(\text{RootSum}\Big[c-\#1^3\&,\cosh(a+b\#1)\text{Chi}\Big(b\Big(\sqrt[3]{c+dx}-\#1\Big)\Big) - \text{Chi}\Big(b\Big(\sqrt[3]{c+dx}-\#1\Big)\Big)\sinh(a+b\#1) - \cosh(a+b\#1)\text{Shi}\Big(b\Big(\sqrt[3]{c+dx}-\#1\Big)\Big) + \sinh(a+b\#1)\text{Shi}\Big(b\Big(\sqrt[3]{c+dx}-\#1\Big)\Big)\&\Big] + \text{RootSum}\Big[c-\#1^3\&,\cosh(a+b\#1)\text{Chi}\Big(b\Big(\sqrt[3]{c+dx}-\#1\Big)\Big) + \text{Chi}\Big(b\Big(\sqrt[3]{c+dx}-\#1\Big)\Big)\sinh(a+b\#1) + \cosh(a+b\#1)\text{Shi}\Big(b\Big(\sqrt[3]{c+dx}-\#1\Big)\Big) + \sinh(a+b\#1)\text{Shi}\Big(b\Big(\sqrt[3]{c+dx}-\#1\Big)\Big)\&\Big]\Big)$$

input
```
Integrate[Cosh[a + b*(c + d*x)^(1/3)]/x,x]
```

output
```
(RootSum[c - #1^3 & , Cosh[a + b*#1]*CoshIntegral[b*((c + d*x)^(1/3) - #1)
] - CoshIntegral[b*((c + d*x)^(1/3) - #1)]*Sinh[a + b*#1] - Cosh[a + b*#1]
*SinhIntegral[b*((c + d*x)^(1/3) - #1)] + Sinh[a + b*#1]*SinhIntegral[b*((
c + d*x)^(1/3) - #1)] & ] + RootSum[c - #1^3 & , Cosh[a + b*#1]*CoshIntegr
al[b*((c + d*x)^(1/3) - #1)] + CoshIntegral[b*((c + d*x)^(1/3) - #1)]*Sinh
[a + b*#1] + Cosh[a + b*#1]*SinhIntegral[b*((c + d*x)^(1/3) - #1)] + Sinh[
a + b*#1]*SinhIntegral[b*((c + d*x)^(1/3) - #1)] & ])/2
```

3.67.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5888, 25, 7267, 5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x}dx$$

$$\Big\downarrow \text{5888}$$

$$\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{dx}d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{dx}d(c+dx)$$

$$\Big\downarrow \text{7267}$$

$$-3\int -\frac{(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{dx}d\sqrt[3]{c+dx}$$

$$\Big\downarrow \text{5816}$$

$$-3\int \left(\frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{3\left(-c+\sqrt[3]{c}-dx\right)}+\frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{3\left(-c-\sqrt[3]{-1}\sqrt[3]{c}-dx\right)}+\frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{3\left(-c+(-1)^{2/3}\sqrt[3]{c}-dx\right)}\right)d\sqrt[3]{c+dx}$$

$$\Big\downarrow \text{2009}$$

$$-3\left(-\frac{1}{3}\cosh\left(a+b\sqrt[3]{c}\right)\text{Chi}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)-\frac{1}{3}\cosh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{Chi}\left(\sqrt[3]{-1}\sqrt[3]{c}b+\sqrt[3]{c+dx}b\right)-\frac{1}{3}\cosh\left(a\right.\right.$$

input
```
Int[Cosh[a + b*(c + d*x)^(1/3)]/x,x]
```

output
```
-3*(-1/3*(Cosh[a + b*c^(1/3)]*CoshIntegral[b*c^(1/3) - b*(c + d*x)^(1/3)])
 - (Cosh[a - (-1)^(1/3)*b*c^(1/3)]*CoshIntegral[(-1)^(1/3)*b*c^(1/3) + b*(
c + d*x)^(1/3)])/3 - (Cosh[a + (-1)^(2/3)*b*c^(1/3)]*CoshIntegral[-((-1)^(
2/3)*b*c^(1/3)) + b*(c + d*x)^(1/3)])/3 + (Sinh[a + b*c^(1/3)]*SinhIntegra
l[b*c^(1/3) - b*(c + d*x)^(1/3)])/3 + (Sinh[a + (-1)^(2/3)*b*c^(1/3)]*Sinh
Integral[(-1)^(2/3)*b*c^(1/3) - b*(c + d*x)^(1/3)])/3 - (Sinh[a - (-1)^(1/
3)*b*c^(1/3)]*SinhIntegral[(-1)^(1/3)*b*c^(1/3) + b*(c + d*x)^(1/3)])/3)
```

3.67. $\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x}dx$

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

rule 5888
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/Coefficient[u, x, 1]^(m + 1)   Subst[Int[(x - Coefficient[u, x
, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p
}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.67.4 Maple [F]

$$\int \frac{\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)}{x}dx$$

input
```
int(cosh(a+b*(d*x+c)^(1/3))/x,x)
```

output
```
int(cosh(a+b*(d*x+c)^(1/3))/x,x)
```

3.67.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(182) = 364$.

Time = 0.27 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.17

$$\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx = \frac{1}{2}\operatorname{Ei}\left(-(dx+c)^{\frac{1}{3}}b - \frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\right)\cosh\left(\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)-a\right)$$
$$+\frac{1}{2}\operatorname{Ei}\left((dx+c)^{\frac{1}{3}}b - \frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\right)\cosh\left(\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)+a\right)$$
$$+\frac{1}{2}\operatorname{Ei}\left(-(dx+c)^{\frac{1}{3}}b + \frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\right)\cosh\left(\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)+a\right)$$
$$+\frac{1}{2}\operatorname{Ei}\left((dx+c)^{\frac{1}{3}}b + \frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\right)\cosh\left(\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)-a\right)$$
$$+\frac{1}{2}\operatorname{Ei}\left(-(dx+c)^{\frac{1}{3}}b+\left(b^3c\right)^{\frac{1}{3}}\right)\cosh\left(a+\left(b^3c\right)^{\frac{1}{3}}\right)$$
$$+\frac{1}{2}\operatorname{Ei}\left((dx+c)^{\frac{1}{3}}b+\left(-b^3c\right)^{\frac{1}{3}}\right)\cosh\left(-a+\left(-b^3c\right)^{\frac{1}{3}}\right)$$
$$+\frac{1}{2}\operatorname{Ei}\left(-(dx+c)^{\frac{1}{3}}b - \frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\right)\sinh\left(\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)-a\right)$$
$$+\frac{1}{2}\operatorname{Ei}\left((dx+c)^{\frac{1}{3}}b - \frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\right)\sinh\left(\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)+a\right)$$
$$-\frac{1}{2}\operatorname{Ei}\left(-(dx+c)^{\frac{1}{3}}b + \frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\right)\sinh\left(\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)+a\right)$$
$$-\frac{1}{2}\operatorname{Ei}\left((dx+c)^{\frac{1}{3}}b + \frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\right)\sinh\left(\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)-a\right)$$
$$-\frac{1}{2}\operatorname{Ei}\left(-(dx+c)^{\frac{1}{3}}b+\left(b^3c\right)^{\frac{1}{3}}\right)\sinh\left(a+\left(b^3c\right)^{\frac{1}{3}}\right)$$
$$-\frac{1}{2}\operatorname{Ei}\left((dx+c)^{\frac{1}{3}}b+\left(-b^3c\right)^{\frac{1}{3}}\right)\sinh\left(-a+\left(-b^3c\right)^{\frac{1}{3}}\right)$$

3.67. $\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx$

input
```
integrate(cosh(a+b*(d*x+c)^(1/3))/x,x, algorithm="fricas")
```

output
```
1/2*Ei(-(d*x + c)^(1/3)*b - 1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(b^
3*c)^(1/3)*(sqrt(-3) + 1) - a) + 1/2*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(
1/3)*(sqrt(-3) + 1))*cosh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1) + a) + 1/2*Ei(
-(d*x + c)^(1/3)*b + 1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(b^3*c)^(1
/3)*(sqrt(-3) - 1) + a) + 1/2*Ei((d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(s
qrt(-3) - 1))*cosh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1) - a) + 1/2*Ei(-(d*x +
 c)^(1/3)*b + (b^3*c)^(1/3))*cosh(a + (b^3*c)^(1/3)) + 1/2*Ei((d*x + c)^(1
/3)*b + (-b^3*c)^(1/3))*cosh(-a + (-b^3*c)^(1/3)) + 1/2*Ei(-(d*x + c)^(1/3
)*b - 1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) +
 1) - a) + 1/2*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1))*s
inh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1) + a) - 1/2*Ei(-(d*x + c)^(1/3)*b + 1
/2*(b^3*c)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1) + a
) - 1/2*Ei((d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1))*sinh(1/2
*(-b^3*c)^(1/3)*(sqrt(-3) - 1) - a) - 1/2*Ei(-(d*x + c)^(1/3)*b + (b^3*c)^
(1/3))*sinh(a + (b^3*c)^(1/3)) - 1/2*Ei((d*x + c)^(1/3)*b + (-b^3*c)^(1/3)
)*sinh(-a + (-b^3*c)^(1/3))
```

3.67.6 Sympy [F]

$$\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx=\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx$$

input
```
integrate(cosh(a+b*(d*x+c)**(1/3))/x,x)
```

output
```
Integral(cosh(a + b*(c + d*x)**(1/3))/x, x)
```

3.67.7 Maxima [F]

$$\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx=\int \frac{\cosh\left(\left(dx+c\right)^{\frac{1}{3}}b+a\right)}{x}\,dx$$

input
```
integrate(cosh(a+b*(d*x+c)^(1/3))/x,x, algorithm="maxima")
```

output
```
integrate(cosh((d*x + c)^(1/3)*b + a)/x, x)
```

3.67.8 Giac [F]

$$\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx=\int \frac{\cosh\left((dx+c)^{\frac{1}{3}}b+a\right)}{x}\,dx$$

input
```
integrate(cosh(a+b*(d*x+c)^(1/3))/x,x, algorithm="giac")
```

output
```
integrate(cosh((d*x + c)^(1/3)*b + a)/x, x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx=\int \frac{\cosh\left(a+b\,(c+d\,x)^{1/3}\right)}{x}\,dx$$

input
```
int(cosh(a + b*(c + d*x)^(1/3))/x,x)
```

output
```
int(cosh(a + b*(c + d*x)^(1/3))/x, x)
```

3.68 $\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx$

3.68.1 Optimal result

Integrand size = 18, antiderivative size = 329

$$\begin{aligned}
\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx
&= -\frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x} + \frac{bd\text{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)\sinh\left(a+b\sqrt[3]{c}\right)}{3c^{2/3}} \\
&\quad - \frac{\sqrt[3]{-1}bd\text{Chi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)\sinh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)}{3c^{2/3}} \\
&\quad + \frac{(-1)^{2/3}bd\text{Chi}\left(-b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)\sinh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)}{3c^{2/3}} \\
&\quad - \frac{bd\cosh\left(a+b\sqrt[3]{c}\right)\text{Shi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}} \\
&\quad - \frac{(-1)^{2/3}bd\cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Shi}\left(b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}} \\
&\quad - \frac{\sqrt[3]{-1}bd\cosh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{Shi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}}
\end{aligned}$$

output
```
-cosh(a+b*(d*x+c)^(1/3))/x-1/3*b*d*cosh(a+b*c^(1/3))*Shi(b*(c^(1/3)-(d*x+c
)^(1/3)))/c^(2/3)-1/3*(-1)^(2/3)*b*d*cosh(a+(-1)^(2/3)*b*c^(1/3))*Shi(b*((
-1)^(2/3)*c^(1/3)-(d*x+c)^(1/3)))/c^(2/3)-1/3*(-1)^(1/3)*b*d*cosh(a-(-1)^(
1/3)*b*c^(1/3))*Shi(b*((-1)^(1/3)*c^(1/3)+(d*x+c)^(1/3)))/c^(2/3)+1/3*b*d*
Chi(b*(c^(1/3)-(d*x+c)^(1/3)))*sinh(a+b*c^(1/3))/c^(2/3)-1/3*(-1)^(1/3)*b*
d*Chi(b*((-1)^(1/3)*c^(1/3)+(d*x+c)^(1/3)))*sinh(a-(-1)^(1/3)*b*c^(1/3))/c
^(2/3)+1/3*(-1)^(2/3)*b*d*Chi(-b*((-1)^(2/3)*c^(1/3)-(d*x+c)^(1/3)))*sinh(
a+(-1)^(2/3)*b*c^(1/3))/c^(2/3)
```

3.68.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.36 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.64

$$\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx = \frac{e^{-a}\left(-3e^{-b\sqrt[3]{c+dx}}\left(1+e^{2\left(a+b\sqrt[3]{c+dx}\right)}\right)+bdx\text{RootSum}\left[c-\#1^3\&,\frac{e^{2a+b\#1}\text{ExpIntegralEi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)}{\#1^2}\&\right.\right.}{}$$

input
```
Integrate[Cosh[a + b*(c + d*x)^(1/3)]/x^2,x]
```

output
```
((-3*(1 + E^(2*(a + b*(c + d*x)^(1/3)))))/E^(b*(c + d*x)^(1/3)) + b*d*x*Ro
otSum[c - #1^3 & , (E^(2*a + b*#1)*ExpIntegralEi[b*((c + d*x)^(1/3) - #1)]
)/#1^2 & ] - b*d*x*RootSum[c - #1^3 & , (Cosh[b*#1]*CoshIntegral[b*((c + d
*x)^(1/3) - #1)] - CoshIntegral[b*((c + d*x)^(1/3) - #1)]*Sinh[b*#1] - Cos
h[b*#1]*SinhIntegral[b*((c + d*x)^(1/3) - #1)] + Sinh[b*#1]*SinhIntegral[b
*((c + d*x)^(1/3) - #1)])/#1^2 & ])/(6*E^a*x)
```

3.68.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5888, 7267, 5812, 5803, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.68. $\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx$

$$\int \frac{\cosh \left(a+b \sqrt[3]{c+dx}\right)}{x^2} \, dx$$

$$\Big\downarrow \ 5888$$

$$d \int \frac{\cosh \left(a+b \sqrt[3]{c+dx}\right)}{d^2 x^2} d(c+dx)$$

$$\Big\downarrow \ 7267$$

$$3d \int \frac{(c+dx)^{2/3} \cosh \left(a+b \sqrt[3]{c+dx}\right)}{d^2 x^2} d\sqrt[3]{c+dx}$$

$$\Big\downarrow \ 5812$$

$$3d \left(-\frac{1}{3} b \int -\frac{\sinh \left(a+b \sqrt[3]{c+dx}\right)}{dx} d\sqrt[3]{c+dx} - \frac{\cosh \left(a+b \sqrt[3]{c+dx}\right)}{3dx}\right)$$

$$\Big\downarrow \ 5803$$

$$3d \left(-\frac{1}{3} b \int \left(\frac{\sinh \left(a+b \sqrt[3]{c+dx}\right)}{3c^{2/3} \left(-c+\sqrt[3]{c}-dx\right)} + \frac{\sinh \left(a+b \sqrt[3]{c+dx}\right)}{3c^{2/3} \left(\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{c+dx}\right)} + \frac{\sinh \left(a+b \sqrt[3]{c+dx}\right)}{3c^{2/3} \left(\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{c+dx}\right)}\right) d\sqrt[3]{c+dx}\right.$$

$$\Big\downarrow \ 2009$$

$$3d \left(-\frac{1}{3} b \left(-\frac{\sinh \left(a+b\sqrt[3]{c}\right) \text{Chi}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)}{3c^{2/3}} + \frac{\sqrt[3]{-1} \sinh \left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right) \text{Chi}\left(\sqrt[3]{-1}\sqrt[3]{c}b+\sqrt[3]{c+dx}b\right)}{3c^{2/3}} - \right.\right.$$

input
```
Int[Cosh[a + b*(c + d*x)^(1/3)]/x^2,x]
```

output
```
3*d*(-1/3*Cosh[a + b*(c + d*x)^(1/3)]/(d*x) - (b*(-1/3*(CoshIntegral[b*c^(
1/3) - b*(c + d*x)^(1/3)]*Sinh[a + b*c^(1/3)])/c^(2/3) + ((-1)^(1/3)*CoshI
ntegral[(-1)^(1/3)*b*c^(1/3) + b*(c + d*x)^(1/3)]*Sinh[a - (-1)^(1/3)*b*c^
(1/3)])/(3*c^(2/3)) - ((-1)^(2/3)*CoshIntegral[-((-1)^(2/3)*b*c^(1/3)) + b
*(c + d*x)^(1/3)]*Sinh[a + (-1)^(2/3)*b*c^(1/3)])/(3*c^(2/3)) + (Cosh[a +
b*c^(1/3)]*SinhIntegral[b*c^(1/3) - b*(c + d*x)^(1/3)])/(3*c^(2/3)) + ((-1
)^(2/3)*Cosh[a + (-1)^(2/3)*b*c^(1/3)]*SinhIntegral[(-1)^(2/3)*b*c^(1/3) -
 b*(c + d*x)^(1/3)])/(3*c^(2/3)) + ((-1)^(1/3)*Cosh[a - (-1)^(1/3)*b*c^(1/
3)]*SinhIntegral[(-1)^(1/3)*b*c^(1/3) + b*(c + d*x)^(1/3)])/(3*c^(2/3))))/
3)
```

3.68. $\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x^2} \, dx$

3.68.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5803
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 5812
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x],
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1,
0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

rule 5888
```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Simp[1/Coefficient[u, x, 1]^(m + 1)   Subst[Int[(x - Coefficient[u, x
, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p
}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.68.4 Maple [F]

$$\int \frac{\cosh\left(a + b(dx + c)^{\frac{1}{3}}\right)}{x^2} dx$$

input
```
int(cosh(a+b*(d*x+c)^(1/3))/x^2,x)
```

output
```
int(cosh(a+b*(d*x+c)^(1/3))/x^2,x)
```

3.68.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(245) = 490$.

Time = 0.29 (sec) , antiderivative size = 706, normalized size of antiderivative = 2.15

$$\int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="fricas")
```

output
```
-1/12*(2*(b^3*c)^(1/3)*d*x*Ei(-(d*x + c)^(1/3)*b + (b^3*c)^(1/3))*cosh(a +
 (b^3*c)^(1/3)) + 2*(-b^3*c)^(1/3)*d*x*Ei((d*x + c)^(1/3)*b + (-b^3*c)^(1/
3))*cosh(-a + (-b^3*c)^(1/3)) - 2*(b^3*c)^(1/3)*d*x*Ei(-(d*x + c)^(1/3)*b
+ (b^3*c)^(1/3))*sinh(a + (b^3*c)^(1/3)) - 2*(-b^3*c)^(1/3)*d*x*Ei((d*x +
c)^(1/3)*b + (-b^3*c)^(1/3))*sinh(-a + (-b^3*c)^(1/3)) - (b^3*c)^(1/3)*(sq
rt(-3)*d*x + d*x)*Ei(-(d*x + c)^(1/3)*b - 1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1)
)*cosh(1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1) - a) - (-b^3*c)^(1/3)*(sqrt(-3)*d*
x + d*x)*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1))*cosh(1/
2*(-b^3*c)^(1/3)*(sqrt(-3) + 1) + a) + (b^3*c)^(1/3)*(sqrt(-3)*d*x - d*x)*
Ei(-(d*x + c)^(1/3)*b + 1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(b^3*c)
^(1/3)*(sqrt(-3) - 1) + a) + (-b^3*c)^(1/3)*(sqrt(-3)*d*x - d*x)*Ei((d*x +
 c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-b^3*c)^(1/3)*(
sqrt(-3) - 1) - a) - (b^3*c)^(1/3)*(sqrt(-3)*d*x + d*x)*Ei(-(d*x + c)^(1/3
)*b - 1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) +
 1) - a) - (-b^3*c)^(1/3)*(sqrt(-3)*d*x + d*x)*Ei((d*x + c)^(1/3)*b - 1/2*
(-b^3*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1) + a)
 - (b^3*c)^(1/3)*(sqrt(-3)*d*x - d*x)*Ei(-(d*x + c)^(1/3)*b + 1/2*(b^3*c)^
(1/3)*(sqrt(-3) - 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1) + a) - (-b^3*c
)^(1/3)*(sqrt(-3)*d*x - d*x)*Ei((d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sq
rt(-3) - 1))*sinh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1) - a) + 12*c*cosh((d...
```

3.68.6 Sympy [F]

$$\int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x^2}\,dx = \int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x^2}\,dx$$

input
```
integrate(cosh(a+b*(d*x+c)**(1/3))/x**2,x)
```

output
```
Integral(cosh(a + b*(c + d*x)**(1/3))/x**2, x)
```

3.68.7 Maxima [F]

$$\int \frac{\cosh \left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx=\int \frac{\cosh \left((dx+c)^{\frac{1}{3}}b+a\right)}{x^2}\,dx$$

input
```
integrate(cosh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="maxima")
```

output
```
integrate(cosh((d*x + c)^(1/3)*b + a)/x^2, x)
```

3.68.8 Giac [F]

$$\int \frac{\cosh \left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx=\int \frac{\cosh \left((dx+c)^{\frac{1}{3}}b+a\right)}{x^2}\,dx$$

input
```
integrate(cosh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="giac")
```

output
```
integrate(cosh((d*x + c)^(1/3)*b + a)/x^2, x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh \left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx=\int \frac{\cosh \left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)}{x^2}\,dx$$

input
```
int(cosh(a + b*(c + d*x)^(1/3))/x^2,x)
```

output
```
int(cosh(a + b*(c + d*x)^(1/3))/x^2, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```